

thm\_2Efrac\_2ENMR  
(TMTrEa7kjGP2kKduBuRMQHafNBHK9dqSYv3)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$ .  
Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 4** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{4}$$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \tag{5}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{6}$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})\ ty\_2Einteger\_2Eint) \tag{7}$$

**Definition 5** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then**  $(the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}$

**Definition 7** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E\_40 (ty$

Let  $c\_2Einteger\_2Eint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)}) \quad (8)$$

**Definition 8** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint$

Let  $ty\_2Efrac\_2Efrac : \iota$  be given. Assume the following.

$$nonempty ty\_2Efrac\_2Efrac \quad (9)$$

Let  $c\_2Efrac\_2Eabs\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Eabs\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod ty\_2Einteger\_2Eint ty\_2Einteger\_2Eint)}) \quad (10)$$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $c\_2Efrac\_2Erep\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Erep\_frac \in ((ty\_2Epair\_2Eprod ty\_2Einteger\_2Eint ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \quad (11)$$

Let  $c\_2Epair\_2EFSST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c\_2Epair\_2EFSST A.27a A.27b \in (A.27a^{(ty\_2Epair\_2Eprod A.27a A.27b)}) \quad (12)$$

**Definition 10** We define  $c\_2Efrac\_2Efrac\_nmr$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap (c\_2Epair\_2EFSST ty$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c\_2Epair\_2EABS\_prod A.27a A.27b \in ((ty\_2Epair\_2Eprod A.27a A.27b)^{((2^{A-27b})^{A-27a})}) \quad (13)$$

**Definition 11** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c\_2$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c\_2Epair\_2ESND A.27a A.27b \in (A.27b^{(ty\_2Epair\_2Eprod A.27a A.27b)}) \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. ((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$\begin{aligned} & ((\forall V0a \in ty\_2Efrac\_2Efrac. ((ap\ c\_2Efrac\_2Eabs\_frac\ ( \\ & \quad ap\ c\_2Efrac\_2Erep\_frac\ V0a) = V0a)) \wedge (\forall V1r \in (ty\_2Epair\_2Eprod \\ & \quad ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint). ((p\ (ap\ (\lambda V2f \in (ty\_2Epair\_2Eprod \\ & \quad ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint). (ap\ (ap\ c\_2Einteger\_2Eint\_lt \\ & \quad (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0))\ (ap\ (c\_2Epair\_2ESND \\ & \quad ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V2f)))\ V1r)) \Leftrightarrow ((ap\ c\_2Efrac\_2Erep\_frac \\ & \quad (ap\ c\_2Efrac\_2Eabs\_frac\ V1r) = V1r)))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \quad \forall V0x \in A.27a. (\forall V1y \in A.27b. ((ap\ (c\_2Epair\_2EFST\ A.27a \\ & \quad A.27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \quad \forall V0x \in A.27a. (\forall V1y \in A.27b. ((ap\ (c\_2Epair\_2ESND\ A.27a \\ & \quad A.27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (20)$$

### Theorem 1

$$\begin{aligned} & (\forall V0a \in ty\_2Einteger\_2Eint. (\forall V1b \in ty\_2Einteger\_2Eint. \\ & \quad ((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_of\_num \\ & \quad c\_2Enum\_2E0))\ V1b)) \Rightarrow ((ap\ c\_2Efrac\_2Efrac\_nmr\ (ap\ c\_2Efrac\_2Eabs\_frac \\ & \quad (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint) \\ & \quad V0a)\ V1b))) = V0a)))) \end{aligned}$$