thm_2Egcd_2EGCD__ADD__R (TMYn9w6Sty6iMBLH1m4QpiUWw7hFZ43RbBD)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Definition 1 *We define* $\mathsf{c\_2Emin\_2E\_3D}$ *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* $\mathsf{c\_2Ebool\_2ET}$ *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* $\mathsf{c\_2Ebool\_2E\_21}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* $\mathsf{c\_2Ebool\_2EF}$ *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* $\mathsf{c\_2Emin\_2E\_3D\_3D\_3E}$ *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* $\mathsf{c\_2Ebool\_2E\_7E}$ *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{3}$$

Definition 7 *We define* $\mathsf{c\_2Ebool\_2E\_2F\_5C}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 13 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Egcd\_2Egcd : \iota$ be given. Assume the following.

$$c\_2Egcd\_2Egcd \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

Definition 14 *We define* c_2Edivides_2Edivides *to be* $\lambda V0a \in ty\_2Enum\_2Enum.\lambda V1b \in ty\_2Enum\_2Enu$

Definition 15 *We define* c_2Egcd_2Eis__gcd *to be* $\lambda V0a \in ty\_2Enum\_2Enum.\lambda V1b \in ty\_2Enum\_2Enum.\lambda$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \quad (9)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))) \quad (10)$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1c \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0a)\ V1c))\ V1c) = V0a))) \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{14}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{15}$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1b \in ty\_2Enum\_2Enum.(\forall V2c \in ty\_2Enum\_2Enum.(\forall V3d \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Egcd\_2Eis\_\_gcd}\ V0a)\ V1b)\ V2c)) \wedge (p\ (ap\ (ap\ (ap\ \mathsf{c\_2Egcd\_2Eis\_\_gcd}\ V0a)\ V1b)\ V3d))) \Rightarrow (V2c = V3d)))))) \tag{16}$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1b \in ty\_2Enum\_2Enum.(\forall V2c \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0a)\ V1b)) \wedge (p\ (ap\ (ap\ (ap\ \mathsf{c\_2Egcd\_2Eis\_\_gcd}\ V0a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1b)\ V0a))\ V2c))) \Rightarrow (p\ (ap\ (ap\ (ap\ \mathsf{c\_2Egcd\_2Eis\_\_gcd}\ V0a)\ V1b)\ V2c)))))) \tag{17}$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1b \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Egcd\_2Eis\_\_gcd}\ V0a)\ V1b)\ (ap\ (ap\ c\_2Egcd\_2Egcd\ V0a)\ V1b))))) \tag{18}$$

Theorem 1

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1b \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Egcd\_2Egcd\ V0a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0a)\ V1b)) = (ap\ (ap\ c\_2Egcd\_2Egcd\ V0a)\ V1b))))$$