

thm_2Ehtrat_2EHRAT__SUCINT
(TMHirssiWXQYfVg7cuaPLPf7FNYYDkJuren)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E3D (2^{A-27a}))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Definition 5 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{6}$$

Let $c_2Ehtrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehtrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})(ty_2Epair_2Eprod\ ty_2Enum_2Enum) \tag{7}$$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 9 We define $c_2Ehrat_2Etrat_1$ to be $(ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty ty_2Ehrat_2Ehrat \quad (9)$$

Let $c_2Ehrat_2Ehrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_ABS_CLASS \in (ty_2Ehrat_2Ehrat^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})}) \quad (10)$$

Definition 10 We define $c_2Ehrat_2Ehrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2$

Definition 11 We define $c_2Ehrat_2Ehrat_1$ to be $(ap c_2Ehrat_2Ehrat_ABS c_2Ehrat_2Etrat_1)$.

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \quad (11)$$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge$ of type $\iota \Rightarrow \iota$).

Definition 13 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap (c_2Emin_2E_40 (ty_2$

Let $c_2Ehrat_2Etrat_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \quad (12)$$

Definition 14 We define $c_2Ehrat_2Ehrat_add$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat.\lambda V1T2 \in ty_2Ehrat_2E$

Let $c_2Ehrat_2Etrat_sucint : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_sucint \in ((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (13)$$

Definition 15 We define $c_2Ehrat_2Ehrat_sucint$ to be $\lambda V0T1 \in ty_2Enum_2Enum.(ap c_2Ehrat_2Ehrat_add$

Definition 16 We define `c_2Equotient_2EEQUIV` to be $\lambda A_{.27a} : \iota. \lambda V0E \in ((2^{A_{.27a}})^{A_{.27a}}). (ap (c_2Ebool_2E$

Definition 17 We define `c_2Ecombin_2EK` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. (\lambda V0x \in A_{.27a}. (\lambda V1y \in A_{.27b}. V0x$

Definition 18 We define `c_2Ecombin_2ES` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda A_{.27c} : \iota. (\lambda V0f \in ((A_{.27c}^{A_{.27b}})^{A_{.27a}})$

Definition 19 We define `c_2Ecombin_2EI` to be $\lambda A_{.27a} : \iota. (ap (ap (c_2Ecombin_2ES A_{.27a} (A_{.27a}^{A_{.27a}}) A_{.27a}$

Definition 20 We define `c_2Equotient_2E_2D_2D_3E` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda A_{.27c} : \iota. \lambda A_{.27d} : \iota. \lambda V0f$

Definition 21 We define `c_2Equotient_2E_3D_3D_3D_3E` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0R1 \in ((2^{A_{.27a}})^{A_{.27a}})$

Definition 22 We define `c_2Equotient_2EQUOTIENT` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0R \in ((2^{A_{.27a}})^{A_{.27a}})$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (14)$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\forall V1y \in A_{.27a}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0p1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\ & (\forall V1p2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\ & (\forall V2q1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\ & (\forall V3q2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\ & (((p (ap (ap c_2Ehrat_2Etrat_eq V0p1) V1p2)) \wedge (p (ap (ap c_2Ehrat_2Etrat_eq V2q1) V3q2))) \Rightarrow (p (ap (ap c_2Ehrat_2Etrat_eq (ap (ap c_2Ehrat_2Etrat_add V0p1) V2q1)) (ap (ap c_2Ehrat_2Etrat_add V1p2) V3q2)))))) \quad (17) \end{aligned}$$

Assume the following.

$$\begin{aligned} & ((p (ap (ap c_2Ehrat_2Etrat_eq (ap c_2Ehrat_2Etrat_succint c_2Enum_2E0)) c_2Ehrat_2Etrat_1)) \wedge (\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Ehrat_2Etrat_eq (ap c_2Ehrat_2Etrat_succint (ap c_2Enum_2ESUC V0n))) (ap (ap c_2Ehrat_2Etrat_add (ap c_2Ehrat_2Etrat_succint V0n)) c_2Ehrat_2Etrat_1)))))) \quad (18) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1q \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& ((p\ (ap\ (ap\ c_2Ehrat_2Etrat_eq\ V0p)\ V1q)) \Leftrightarrow ((ap\ c_2Ehrat_2Etrat_eq \\
& \quad V0p) = (ap\ c_2Ehrat_2Etrat_eq\ V1q))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& ty_2Enum_2Enum)\ ty_2Ehrat_2Ehrat)\ c_2Ehrat_2Etrat_eq)\ c_2Ehrat_2Ehrat_ABS) \\
& \quad c_2Ehrat_2Ehrat_REP))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& A_27a\ A_27a)\ (c_2Emin_2E_3D\ A_27a))\ (c_2Ecombin_2EI\ A_27a))\ (\\
& \quad c_2Ecombin_2EI\ A_27a)))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& ((\forall V1x \in A_27a. (\forall V2y \in A_27a. ((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Leftrightarrow \\
& ((ap\ V0R\ V1x) = (ap\ V0R\ V2y)))))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ (ap\ V0R \\
& \quad V3x)\ V3x))) \wedge ((\forall V4x \in A_27a. (\forall V5y \in A_27a. ((p\ (ap\ (\\
& ap\ V0R\ V4x)\ V5y)) \Rightarrow (p\ (ap\ (ap\ V0R\ V5y)\ V4x)))))) \wedge (\forall V6x \in A_27a. \\
& (\forall V7y \in A_27a. (\forall V8z \in A_27a. (((p\ (ap\ (ap\ V0R\ V6x)\ V7y)) \wedge \\
& (p\ (ap\ (ap\ V0R\ V7y)\ V8z))) \Rightarrow (p\ (ap\ (ap\ V0R\ V6x)\ V8z))))))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}). (\forall V1abs1 \in (A_27c^{A_27a}). (\forall V2rep1 \in \\
& (A_27a^{A_27c}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}). (\forall V4abs2 \in \\
& (A_27d^{A_27b}). (\forall V5rep2 \in (A_27b^{A_27d}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& (A_27b^{A_27a})\ (A_27d^{A_27c}))\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\
& A_27a\ A_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27c \\
& A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& A_27a\ A_27d\ A_27c\ A_27b)\ V1abs1)\ V5rep2))))))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x \in A_27b.(\forall V4y \in \\
& \quad A_27b.((V3x = V4y) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3x))\ (ap\ V2rep\ V4y)))))))))) \\
& \hspace{15em} (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& \quad A_27a.(\forall V5y1 \in A_27a.(\forall V6y2 \in A_27a.(((p\ (ap\ (ap\ V0R \\
& \quad V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2)))))))))) \\
& \hspace{15em} (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0REL \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& \quad A_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2)))))))))) \\
& \hspace{15em} (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).(\forall V8x \in A_27a.(\forall V9y \in \\
& \quad A_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y)))))))))) \\
& \hspace{15em} (27)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (((ap\ c_2Ehrat_2Ehrat_sucint\ c_2Enum_2E0) = c_2Ehrat_2Ehrat_1) \wedge \\
& \quad (\forall V0n \in ty_2Enum_2Enum.((ap\ c_2Ehrat_2Ehrat_sucint\ (\\
& \quad ap\ c_2Enum_2ESUC\ V0n)) = (ap\ (ap\ c_2Ehrat_2Ehrat_add\ (ap\ c_2Ehrat_2Ehrat_sucint \\
& \quad V0n))\ c_2Ehrat_2Ehrat_1))))
\end{aligned}$$