

thm_2Ehtrat_2ETRAT__ADD__SYM__EQ
(TMFdfG1Xxh6DhAX1PdQcze2hZ7cjZMKoZiZ)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num ($

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{5}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A \\
& V1n) V0m)))) \\
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1y \in ty_2Enum_2Enum. (\\
& \forall V2x_27 \in ty_2Enum_2Enum. (\forall V3y_27 \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Ehrat_2Etrat_add (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V0x) V1y)) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V2x_27) V3y_27)) = (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0x)) (ap c_2Enum_2ESUC \\
& V3y_27))) (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V2x_27)) \\
& (ap c_2Enum_2ESUC V1y)))))) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2A \\
& (ap c_2Enum_2ESUC V1y)) (ap c_2Enum_2ESUC V3y_27))))))))) \\
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& A_27b. (((ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y) = (ap (ap \\
& (c_2Epair_2E_2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0x \in (ty_2Epair_2Eprod A_27a A_27b). ((ap (ap (c_2Epair_2E_2C \\
& A_27a A_27b) (ap (c_2Epair_2E_2C A_27a A_27b) V0x)) (ap (c_2Epair_2ESND \\
& A_27a A_27b) V0x)) = V0x)) \\
\end{aligned} \tag{17}$$

Theorem 1

$$\begin{aligned}
& (\forall V0h \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& (\forall V1i \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& ((ap (ap c_2Ehrat_2Etrat_add V0h) V1i) = (ap (ap c_2Ehrat_2Etrat_add \\
& V1i) V0h))))
\end{aligned}$$