

thm_2Ehrat_2ETRAT__ADD__TOTAL
 (TMVE8KSFvEMALZ83Xa7dPVJYJZnEHgoCL1e)

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Let $c_2Enum_2ZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2ABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2ABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be ($ap\ c_2Enum_2ABS_num\ c_2Enum_2ZERO_REP$).

Definition 3 We define $c_2Earithmetic_2ZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2SUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2SUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 4 We define c_2Ebool_2ET to be ($ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$)

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x)))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2ABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2EBIT1\ n)\ V)$

Definition 8 We define $c_2Earthmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 9 We define c_2 to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_{\text{2Ebool_2E_3F}}$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\;V0P\;(ap\;(c_{\text{2Emin_2E_40}}\;$

Definition 11 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_{\text{min_inj_o}}$ to be $\lambda P \in 2.\lambda Q \in 2.\text{inj_o} (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E))$

Definition 14 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 16 We define $c_{\text{c_Ebool_2E_5C_2F}}$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_{\text{c_Ebool_2E_21}}\ 2)\ (\lambda V2t \in$

$$\forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty} (\text{ty_2Epair_2Eprod } A0\ A1) \quad (7)$$

Let $c_2Ehrat_2Etrat_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^8)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2 Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) (ty_2Epair_2Eprod\ ty_2Enum_2L)) \quad (10)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{\text{27a}}.\text{nonempty } A_{\text{27a}} \Rightarrow \forall A_{\text{27b}}.\text{nonempty } A_{\text{27b}} \Rightarrow c_{\text{2Epair_2ESND}}(A_{\text{27a}}, A_{\text{27b}}) \in (A_{\text{27b}}^{(\text{ty_2Epair_2Eprod } A_{\text{27a}} \ A_{\text{27b}})}) \quad (11)$$

Let $c_{-ZEPall-ZETSI} : \iota \rightarrow \iota \rightarrow \iota$ be given. Assume the following.

$$\forall A \in \text{nonempty } A \Rightarrow \forall A \in \text{nonempty } A \Rightarrow \exists E \in \text{pair_2E} \ F \in \text{S1} \\ A \in \text{27a} \quad A \in \text{27b} \in (A \in \text{27a}^{(ty_2E \text{pair_2Eprod } A \in \text{27a} \quad A \in \text{27b})}) \quad (12)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{\text{27a}}.\text{nonempty } A_{\text{27a}} \Rightarrow \forall A_{\text{27b}}.\text{nonempty } A_{\text{27b}} \Rightarrow c_{\text{2Epair_2EABS_prod}}(A_{\text{27a}}, A_{\text{27b}}) \in ((ty_{\text{2Epair_2Eprod}}(A_{\text{27a}}, A_{\text{27b}}))^{((2^{A_{\text{27b}}})^{A_{\text{27a}}})}) \quad (13)$$

Definition 17 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2$

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap (c_2$

Definition 19 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2E$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))) \\ & (14) \end{aligned}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum. (V0m = (ap c_2Enum_2ESUC V1n)))))) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0m) = (ap (ap \\ & c_2Earithmetic_2E_2B V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))))) \\ & (16) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\ & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\ & (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\ & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\ & V0m) V1n))))))) \\ & (17) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A \\ & V1n) V0m)))))) \\ & (18) \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (ap \\
 & \quad \quad (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2B \\
 & \quad \quad (ap (ap c_2Earithmetic_2E_2A V0m) V2p)) (ap (ap c_2Earithmetic_2E_2A \\
 & \quad \quad V1n) V2p)))))))
 \end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V0m) \\
 & \quad (ap (ap c_2Earithmetic_2E_2A V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2A \\
 & \quad (ap (ap c_2Earithmetic_2E_2A V0m) V1n) V2p)))))))
 \end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \quad (p (ap (ap c_2Eprim_rec_2E_3C V1n) V0m)) \Rightarrow (\exists V2p \in ty_2Enum_2Enum. \\
 & \quad V0m = (ap (ap c_2Earithmetic_2E_2B V1n) (ap (ap c_2Earithmetic_2E_2B \\
 & \quad V2p) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
 & \quad c_2Earithmetic_2EZERO)))))))
 \end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \quad (V0m = V1n) \vee ((p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \vee (p (ap (ap \\
 & \quad c_2Eprim_rec_2E_3C V1n) V0m)))))))
 \end{aligned} \tag{22}$$

Assume the following.

$$True \tag{23}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
 V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{25}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
 & \quad (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
 & \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
 \end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
 & \quad (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
 & \quad (p V0t)) \Leftrightarrow (p V0t))))))
 \end{aligned} \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (28)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (29)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Enum_2Enum.(\forall V1y \in ty_2Enum_2Enum.(\forall V2x_27 \in ty_2Enum_2Enum.(\forall V3y_27 \in ty_2Enum_2Enum. \\ & ((ap (ap c_2Ehrat_2Etrat_add (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V0x) V1y)) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V2x_27) V3y_27)) = (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0x)) (ap c_2Enum_2ESUC V3y_27))) (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V2x_27)) (ap (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V1y)))))) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V1y)) (ap c_2Enum_2ESUC V3y_27))))))))))) \quad (33) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Enum_2Enum.(\forall V1y \in ty_2Enum_2Enum.(\forall V2x_27 \in ty_2Enum_2Enum.(\forall V3y_27 \in ty_2Enum_2Enum. \\ & ((p (ap (ap c_2Ehrat_2Etrat_eq (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V0x) V1y)) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V2x_27) V3y_27))) \Leftrightarrow ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0x)) (ap c_2Enum_2ESUC V3y_27)) = (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V2x_27)) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V1y))))))))))) \quad (34) \end{aligned}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). ((ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ A_27b)\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V0x))\ (ap\ (c_2Epair_2ESND \\ & A_27a\ A_27b)\ V0x)) = V0x)) \end{aligned} \quad (36)$$

Assume the following.

$$(((ap\ c_2Eprim_rec_2EPRE\ c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V0m \in ty_2Enum_2Enum. ((ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Enum_2ESUC\ V0m)) = V0m))) \quad (37)$$

Theorem 1

$$\begin{aligned} & (\forall V0h \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). \\ & \quad (\forall V1i \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). \\ & \quad ((p\ (ap\ (ap\ c_2Ehrat_2Etrat_eq\ V0h)\ V1i)) \vee ((\exists V2d \in (ty_2Epair_2Eprod \\ & \quad ty_2Enum_2Enum\ ty_2Enum_2Enum). (p\ (ap\ (ap\ c_2Ehrat_2Etrat_eq \\ & \quad V0h)\ (ap\ (ap\ c_2Ehrat_2Etrat_add\ V1i)\ V2d)))) \vee (\exists V3d \in (\\ & \quad ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). (p\ (ap\ (ap\ c_2Ehrat_2Etrat_eq \\ & \quad V1i)\ (ap\ (ap\ c_2Ehrat_2Etrat_add\ V0h)\ V3d)))))))))) \end{aligned}$$