thm_2Ehrat_2ETRAT__ADD__WELLDEFINED2 (TMPawnM-Sas8AC1bARKmzPF5bBCnpuZMENKR)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $c\_2Ehrat\_2Etrat\_\_add : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty} \tag{3}$$

Let $c\_2Ehrat\_2Etrat\_\_eq : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \tag{4}$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \quad (6)$$

Assume the following.

$$(\forall V0p \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). (\forall V1q \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). (\forall V2r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). (((p\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_eq\ V0p)\ V1q)) \wedge (p\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_eq\ V1q)\ V2r))) \Rightarrow (p\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_eq\ V0p)\ V2r)))))) \quad (7)$$

Assume the following.

$$(\forall V0h \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). (\forall V1i \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). ((ap\ (ap\ c\_2Ehrat\_2Etrat\_\_add\ V0h)\ V1i) = (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_add\ V1i)\ V0h)))) \quad (8)$$

Assume the following.

$$(\forall V0p \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). (\forall V1q \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). (\forall V2r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). ((p\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_eq\ V0p)\ V1q)) \Rightarrow (p\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_eq\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_add\ V0p)\ V2r))\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_add\ V1q)\ V2r))))))) \quad (9)$$

Theorem 1

$$(\forall V0p1 \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). (\forall V1p2 \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). (\forall V2q1 \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). (\forall V3q2 \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). (((p\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_eq\ V0p1)\ V1p2)) \wedge (p\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_eq\ V2q1)\ V3q2))) \Rightarrow (p\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_eq\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_add\ V0p1)\ V2q1))\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_\_add\ V1p2)\ V3q2))))))))$$