

thm\_2Ehrat\_2ETRAT\_\_EQ\_\_REFL  
(TMQmxw2oQNEsj1LNPeWrU4r6u3HWb6a6zCB)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P))$

**Definition 4** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (ap (ap (c\_2Emin\_2E\_3D (2^{27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x))$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{5}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{6}$$

Let  $c\_2Ehrat\_2Etrat\_eq : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (7)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (8)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (9)$$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b} A\_27a)}) \quad (10)$$

**Definition 7** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)) = V0x)$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.(\forall V2x\_27 \in ty\_2Enum\_2Enum.(\forall V3y\_27 \in ty\_2Enum\_2Enum. \\ & ((p\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_eq\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V0x)\ V1y))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V2x\_27)\ V3y\_27))) \Leftrightarrow ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Enum\_2ESUC\ V0x))\ (ap\ c\_2Enum\_2ESUC\ V3y\_27)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Enum\_2ESUC\ V2x\_27))\ (ap\ c\_2Enum\_2ESUC\ V1y)))))))) \end{aligned} \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).((ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V0x))\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V0x)) = V0x)) \quad (12)$$

**Theorem 1**

$$(\forall V0p \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). (p\ (ap\ (ap\ c\_2Ehrat\_2Etrat\_eq\ V0p)\ V0p)))$$