

thm_2Ehrat_2ETRAT_LDISTRI (TMa8gQbE8VoneED5gUHK2wFvDe4qKmVmxDR)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c_2Emin_2E_40 A$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 4 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 5 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 6 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ 0)$

Let c_2 be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (6)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap (ap c_2Earithmetic_2EBIT1 n) 0) n))$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 11 We define $c_{\text{2Emin_3D_3D_3E}}$ to be $\lambda P \in 2.\lambda Q \in 2.\text{inj_o} (p \Rightarrow p Q)$ of type ι .

Definition 12 We define $c_{_2Ebool_2E_5C_2F}$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_{_2Ebool_2E_21}\ 2)\ (\lambda V2t \in$

Definition 13 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty}(\text{ty_2Epair_2Eprod } A0\ A1) \quad (7)$$

Let $c_2Ehrat_2Etrat_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{\text{ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum}})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \quad (8)$$

Let $c_2Ehrat_2Etrat_mul : \iota$ be given. Assume the following.

$c_2Ehrat_2Etrat_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{\omega})^{\omega}$

Let c_2 be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (10)$$

Let $c_{Ehrat_Etrat_eq} : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)} \quad (11)$$

Definition 14 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_Emin_2E_3D_3D_3E\ V0t)\ c_Ebool_2E))$

Let $c_{\text{2Epair_2ESND}} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epair_2ESND \\ A_27a \ A_27b \in (A_27b^{(ty_2Epair_2Eprod \ A_27a \ A_27b)}) \quad (12)$$

Let $c : \text{Epair} \rightarrow \text{EFST} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{_27a}.nonempty\ A_{_27a} \Rightarrow \forall A_{_27b}.nonempty\ A_{_27b} \Rightarrow c_{_2Epair_2EFST}(A_{_27a}\ A_{_27b}) \in (A_{_27a}^{(ty_2Epair_2Eprod\ A_{_27a}\ A_{_27b})}) \quad (13)$$

Definition 15 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{\text{27}}.a.\text{nonempty } A_{\text{27}}a \Rightarrow \forall A_{\text{27}}.b.\text{nonempty } A_{\text{27}}b \Rightarrow c_{\text{2Epair_2EABS_prod}}(A_{\text{27}}a, A_{\text{27}}b) \in ((ty_{\text{2Epair_2Eprod}}(A_{\text{27}}a, A_{\text{27}}b))^{((2^{A_{\text{27}}b})^{A_{\text{27}}a})}) \quad (14)$$

Definition 16 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Definition 17 We define $c_{\text{Ebool_ECOND}}$ to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

Definition 18 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (ap (c_2Ebool_2B$

Assume the following.

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum. (V0m = (ap\ c_2Enum_2ESUC\ V1n))))) \quad (16)$$

Assume the following.

$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL
(ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL
(ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap
(ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) =
(ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n))))))))))$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A V1n) V0m)))) \quad (18)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (ap \\
 & \quad \quad (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2B \\
 & \quad \quad (ap (ap c_2Earithmetic_2E_2A V0m) V2p)) (ap (ap c_2Earithmetic_2E_2A \\
 & \quad \quad V1n) V2p)))))))
 \end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V2p) \\
 & \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) = (ap (ap c_2Earithmetic_2E_2B \\
 & \quad (ap (ap c_2Earithmetic_2E_2A V2p) V0m)) (ap (ap c_2Earithmetic_2E_2A \\
 & \quad V2p) V1n)))))))
 \end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V0m) \\
 & \quad (ap (ap c_2Earithmetic_2E_2A V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2A \\
 & \quad (ap (ap c_2Earithmetic_2E_2A V0m) V1n) V2p)))))))
 \end{aligned} \tag{21}$$

Assume the following.

$$True \tag{22}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{24}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
 & \quad (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
 & \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
 \end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
 & \quad (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
 & \quad (p V0t)) \Leftrightarrow (p V0t))))))
 \end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
 & \quad True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
 & \quad (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))
 \end{aligned} \tag{27}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (28)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Enum_2Enum.(\forall V1y \in ty_2Enum_2Enum.(\forall V2x_27 \in ty_2Enum_2Enum.(\forall V3y_27 \in ty_2Enum_2Enum. \\ & ((ap (ap c_2Ehrat_2Etrat_add (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V0x) V1y)) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V2x_27) V3y_27)) = (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0x)) (ap c_2Enum_2ESUC V3y_27))) (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V2x_27)) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V1y)))))) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V1y)) (ap c_2Enum_2ESUC V3y_27))))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Enum_2Enum.(\forall V1y \in ty_2Enum_2Enum.(\forall V2x_27 \in ty_2Enum_2Enum.(\forall V3y_27 \in ty_2Enum_2Enum. \\ & ((ap (ap c_2Ehrat_2Etrat_mul (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V0x) V1y)) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V2x_27) V3y_27)) = (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0x)) (ap c_2Enum_2ESUC V2x_27)))) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V1y)) (ap c_2Enum_2ESUC V3y_27))))))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Enum_2Enum.(\forall V1y \in ty_2Enum_2Enum.(\forall V2x_27 \in ty_2Enum_2Enum.(\forall V3y_27 \in ty_2Enum_2Enum. \\ & ((p (ap (ap c_2Ehrat_2Etrat_eq (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V0x) V1y)) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) V2x_27) V3y_27))) \Leftrightarrow ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0x)) (ap c_2Enum_2ESUC V3y_27)) = (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V2x_27)) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V1y))))))))))) \end{aligned} \quad (33)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap c_2Enum_2ESUC V0n) = c_2Enum_2E0))) \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod A_27a A_27b). ((ap (ap (c_2Epair_2E_2C \\ & A_27a A_27b) (ap (c_2Epair_2EFST A_27a A_27b) V0x)) (ap (c_2Epair_2ESND \\ & A_27a A_27b) V0x)) = V0x)) \end{aligned} \quad (35)$$

Assume the following.

$$(((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V0m \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE (ap c_2Enum_2ESUC V0m)) = V0m))) \quad (36)$$

Theorem 1

$$\begin{aligned} & (\forall V0h \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\ & (\forall V1i \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\ & (\forall V2j \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\ & (p (ap (ap c_2Ehrat_2Etrat_eq (ap (ap c_2Ehrat_2Etrat_mul V0h) \\ & (ap (ap c_2Ehrat_2Etrat_add V1i) V2j))) (ap (ap c_2Ehrat_2Etrat_add \\ & (ap (ap c_2Ehrat_2Etrat_mul V0h) V1i)) (ap (ap c_2Ehrat_2Etrat_mul \\ & V0h) V2j))))))) \end{aligned}$$