

thm_2Ehrat_2ETRAT__MUL__LINV (TMTnzGz-FavPesiXGD7RHSWjua5D3rWXL9Xt)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota))$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y) \text{ of type } \iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c_2Emin_2E_40 A) (c_2Emin_2E_3D A))))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (1)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (3)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (4)$$

Definition 4 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 5 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (6)$$

Definition 17 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E))$.
 Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a &\Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epair_2ESND \\ A_27a \ A_27b &\in (A_27b(ty_2Epair_2Eprod \ A_27a \ A_27b)) \end{aligned} \quad (13)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a &\Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epair_2EFST \\ A_27a \ A_27b &\in (A_27a(ty_2Epair_2Eprod \ A_27a \ A_27b)) \end{aligned} \quad (14)$$

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\dots)))$.

Definition 19 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E)))$.
 Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(&((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2B (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))) \end{aligned} \quad (15)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum.(V0m = (ap c_2Enum_2ESUC V1n))))) \quad (16)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(&((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n))))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(&(ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A V1n) V0m)))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V0m) \\
 & \quad (ap (ap c_2Earithmetic_2E_2A V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2A \\
 & \quad (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V2p)))))) \\
 \end{aligned} \tag{19}$$

Assume the following.

$$True \tag{20}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{22}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
 & \quad (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
 & \quad (p V0t)) \Leftrightarrow (p V0t)))))) \\
 \end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
 & \quad True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
 & \quad (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \\
 \end{aligned} \tag{24}$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
 ((\neg False) \Leftrightarrow True)))) \tag{25}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{26}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
 & \quad (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\
 & \quad (p V0t)))))) \\
 \end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Enum_2Enum. (\forall V1y \in ty_2Enum_2Enum. \\
 & \quad (ap c_2Ehrat_2Etrat_inv (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
 & \quad ty_2Enum_2Enum) V0x) V1y)) = (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
 & \quad ty_2Enum_2Enum) V1y) V0x)))) \\
 \end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1y \in ty_2Enum_2Enum. (\forall V2x_27 \in ty_2Enum_2Enum. (\forall V3y_27 \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Ehrat_2Etrat_mul (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V0x) V1y)) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V2x_27) V3y_27)) = (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) (ap c_2Eprim_rec_2EPRE (ap (ap c_2Earithmetic_2E_2A \\
& (ap c_2Enum_2ESUC V0x)) (ap c_2Enum_2ESUC V2x_27)))) (ap c_2Eprim_rec_2EPRE \\
& (ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V1y)) (ap c_2Enum_2ESUC \\
& V3y_27)))))))))) \\
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1y \in ty_2Enum_2Enum. (\forall V2x_27 \in ty_2Enum_2Enum. (\forall V3y_27 \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Ehrat_2Etrat_eq (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V0x) V1y)) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V2x_27) V3y_27))) \Leftrightarrow ((ap (ap c_2Earithmetic_2E_2A \\
& (ap c_2Enum_2ESUC V0x)) (ap c_2Enum_2ESUC V3y_27)) = (ap (ap c_2Earithmetic_2E_2A \\
& (ap c_2Enum_2ESUC V2x_27)) (ap c_2Enum_2ESUC V1y)))))) \\
\end{aligned} \tag{30}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap c_2Enum_2ESUC V0n) = c_2Enum_2E0))) \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \\
& \forall V0x \in (ty_2Epair_2Eprod A_27a A_27b). ((ap (ap (c_2Epair_2E_2C \\
& A_27a A_27b) (ap (c_2Epair_2EFST A_27a A_27b) V0x)) (ap (c_2Epair_2ESND \\
& A_27a A_27b) V0x)) = V0x)) \\
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V0m \in \\
& ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE (ap c_2Enum_2ESUC V0m)) = \\
& V0m))) \\
\end{aligned} \tag{33}$$

Theorem 1

$$\begin{aligned}
& (\forall V0h \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& (p (ap (ap c_2Ehrat_2Etrat_eq (ap (ap c_2Ehrat_2Etrat_mul (ap \\
& c_2Ehrat_2Etrat_inv V0h)) V0h)) c_2Ehrat_2Etrat_1))) \\
\end{aligned}$$