

# thm\_2Ehreal\_2ECUT\_NONEMPTY (TMdYff57DggnVQd6mFAhDv5oFYmMiFVeBGW)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2ET` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A-27a}))))$

**Definition 4** We define `c_2Ebool_2EF` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V0t \in 2.V0t))$ .

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$\text{nonempty } \text{ty\_2Enum\_2Enum} \tag{1}$$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty\_2Epair\_2Eprod } A0 \ A1) \tag{2}$$

Let `ty_2Ehrat_2Ehrat` :  $\iota$  be given. Assume the following.

$$\text{nonempty } \text{ty\_2Ehrat\_2Ehrat} \tag{3}$$

Let `c_2Ehrat_2Ehrat__REP__CLASS` :  $\iota$  be given. Assume the following.

$$\text{c\_2Ehrat\_2Ehrat\_REP\_CLASS} \in ((2^{(\text{ty\_2Epair\_2Eprod } \text{ty\_2Enum\_2Enum } \text{ty\_2Enum\_2Enum})})^{\text{ty\_2Ehrat\_2Ehrat}}) \tag{4}$$

**Definition 5** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P \ x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge P \ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 6** We define `c_2Ehrat_2Ehrat__REP` to be  $\lambda V0a \in \text{ty\_2Ehrat\_2Ehrat}. (\text{ap } (\text{c\_2Emin\_2E\_40 } (\text{ty\_2Ehrat\_2Ehrat } V0a)))$

Let `c_2Ehrat_2Etratr__add` :  $\iota$  be given. Assume the following.

$$\text{c\_2Ehrat\_2Etratr\_add} \in (((\text{ty\_2Epair\_2Eprod } \text{ty\_2Enum\_2Enum } \text{ty\_2Enum\_2Enum})^{\text{ty\_2Epair\_2Eprod } \text{ty\_2Enum\_2Enum}}))^{\text{ty\_2Ehrat\_2Ehrat}} \tag{5}$$

Let  $c\_2Ehrat\_2Etrat\_eq : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (6)$$

Let  $c\_2Ehrat\_2Ehtrat\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Ehtrat\_ABS\_CLASS \in (ty\_2Ehrat\_2Ehtrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \quad (7)$$

**Definition 7** We define  $c\_2Ehrat\_2Ehtrat\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$

**Definition 8** We define  $c\_2Ehrat\_2Ehtrat\_add$  to be  $\lambda V0T1 \in ty\_2Ehrat\_2Ehtrat.\lambda V1T2 \in ty\_2Ehrat\_2Ehtrat$

**Definition 9** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a))))$

**Definition 10** We define  $c\_2Ehreal\_2Ehtrat\_lt$  to be  $\lambda V0x \in ty\_2Ehrat\_2Ehtrat.\lambda V1y \in ty\_2Ehrat\_2Ehtrat$

**Definition 11** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 12** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E))$

**Definition 13** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.)))))$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (8)$$

Let  $c\_2Ehreal\_2Ecut : \iota$  be given. Assume the following.

$$c\_2Ehreal\_2Ecut \in ((2^{ty\_2Ehreal\_2Ehreal})^{ty\_2Ehreal\_2Ehreal}) \quad (9)$$

**Definition 14** We define  $c\_2Ehreal\_2Eisacut$  to be  $\lambda V0C \in (2^{ty\_2Ehreal\_2Ehreal}).(ap\ (ap\ c\_2Ebool\_2E\_2F\_5C\ C))$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(p\ (ap\ c\_2Ehreal\_2Eisacut\ (ap\ c\_2Ehreal\_2Ecut\ V0X)))) \quad (13)$$

**Theorem 1**

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\exists V1x \in ty\_2Ehtrat\_2Ehtrat.(p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V1x))))$$