

thm_2Ehreal_2EEQUAL_CUTS (TMYdQr- BXx6eaeEYrMBXoKDVxdrnNL3AnEZ6)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat \tag{3}$$

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})\ ty_2Ehrat_2Ehrat) \tag{4}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap (c_2Emin_2E_40 (ty_2Ehrat_2Ehrat_REP_CLASS V0a)))$

Let $c_2Ehrat_2Etrat_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})) \quad (5)$$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})(ty_2Epair_2Eprod\ ty_2Enum_2Enum)) \quad (6)$$

Let $c_2Ehrat_2Ehtrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehtrat_ABS_CLASS \in (ty_2Ehtrat_2Ehtrat^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}))}) \quad (7)$$

Definition 9 We define $c_2Ehtrat_2Ehtrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 10 We define $c_2Ehtrat_2Ehtrat_add$ to be $\lambda V0T1 \in ty_2Ehtrat_2Ehtrat.\lambda V1T2 \in ty_2Ehtrat_2Ehtrat$

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 12 We define $c_2Ehreal_2Ehtrat_lt$ to be $\lambda V0x \in ty_2Ehtrat_2Ehtrat.\lambda V1y \in ty_2Ehtrat_2Ehtrat$

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 14 We define $c_2Ehreal_2Eisacut$ to be $\lambda V0C \in (2^{ty_2Ehtrat_2Ehtrat}).(ap\ (ap\ c_2Ebool_2E_2F_5C\ ($

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (8)$$

Let $c_2Ehreal_2Ecut : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ecut \in ((2^{ty_2Ehtrat_2Ehtrat})^{ty_2Ehreal_2Ehreal}) \quad (9)$$

Let $c_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ehreal \in (ty_2Ehreal_2Ehreal^{(2^{ty_2Ehtrat_2Ehtrat})}) \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & ((\forall V0a \in ty_2Ehreal_2Ehreal.((ap\ c_2Ehreal_2Ehreal\ (ap \\ & c_2Ehreal_2Ecut\ V0a)) = V0a)) \wedge (\forall V1r \in (2^{ty_2Ehtrat_2Ehtrat}). \\ & ((p\ (ap\ c_2Ehreal_2Eisacut\ V1r)) \Leftrightarrow ((ap\ c_2Ehreal_2Ecut\ (ap\ c_2Ehreal_2Ehreal \\ & V1r)) = V1r)))) \end{aligned} \quad (13)$$

Theorem 1

$$\begin{aligned} & (\forall V0X \in ty_2Ehreal_2Ehreal.(\forall V1Y \in ty_2Ehreal_2Ehreal. \\ & (((ap\ c_2Ehreal_2Ecut\ V0X) = (ap\ c_2Ehreal_2Ecut\ V1Y)) \Rightarrow (V0X = V1Y)))) \end{aligned}$$