

thm_2Ehreal_2EHRAT__INV__MUL (TM- NtK5reE72Q65zVPgNepG7W4zAeweWaShm)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 8 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{4}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{5}$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2E$

Definition 10 We define $c_2Ehrat_2Etrat_1$ to be $(ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (6)$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty ty_2Ehrat_2Ehrat \quad (7)$$

Let $c_2Ehrat_2Ehrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_ABS_CLASS \in (ty_2Ehrat_2Ehrat^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})}) \quad (8)$$

Definition 11 We define $c_2Ehrat_2Ehrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum$

Definition 12 We define $c_2Ehrat_2Ehrat_1$ to be $(ap c_2Ehrat_2Ehrat_ABS c_2Ehrat_2Etrat_1)$.

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \quad (9)$$

Definition 13 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap (c_2Emin_2E_40 (ty_2$

Let $c_2Ehrat_2Etrat_inv : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_inv \in ((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (10)$$

Definition 15 We define $c_2Ehrat_2Ehrat_inv$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat.(ap c_2Ehrat_2Ehrat_ABS$

Let $c_2Ehrat_2Etrat_mul : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (11)$$

Definition 16 We define $c_2Ehrat_2Ehrat_mul$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat.\lambda V1T2 \in ty_2Ehrat_2E$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (14)$$

Assume the following.

$$(\forall V0h \in ty_2Ehrt_2Ehrt. (\forall V1i \in ty_2Ehrt_2Ehrt. ((ap (ap c_2Ehrt_2Ehrt_mul V0h) V1i) = (ap (ap c_2Ehrt_2Ehrt_mul V1i) V0h)))) \quad (15)$$

Assume the following.

$$(\forall V0h \in ty_2Ehrt_2Ehrt. (\forall V1i \in ty_2Ehrt_2Ehrt. (\forall V2j \in ty_2Ehrt_2Ehrt. ((ap (ap c_2Ehrt_2Ehrt_mul V0h) (ap (ap c_2Ehrt_2Ehrt_mul V1i) V2j)) = (ap (ap c_2Ehrt_2Ehrt_mul (ap (ap c_2Ehrt_2Ehrt_mul V0h) V1i)) V2j)))))) \quad (16)$$

Assume the following.

$$(\forall V0h \in ty_2Ehrt_2Ehrt. ((ap (ap c_2Ehrt_2Ehrt_mul c_2Ehrt_2Ehrt_1) V0h) = V0h)) \quad (17)$$

Assume the following.

$$(\forall V0x \in ty_2Ehrt_2Ehrt. ((ap (ap c_2Ehrt_2Ehrt_mul V0x) (ap c_2Ehrt_2Ehrt_inv V0x)) = c_2Ehrt_2Ehrt_1)) \quad (18)$$

Assume the following.

$$(\forall V0x \in ty_2Ehrt_2Ehrt. (\forall V1y \in ty_2Ehrt_2Ehrt. (\forall V2z \in ty_2Ehrt_2Ehrt. (((ap (ap c_2Ehrt_2Ehrt_mul V0x) V2z)) \Leftrightarrow (V1y = V2z)))))) \quad (19)$$

Theorem 1

$$(\forall V0x \in ty_2Ehrt_2Ehrt. (\forall V1y \in ty_2Ehrt_2Ehrt. ((ap c_2Ehrt_2Ehrt_inv (ap (ap c_2Ehrt_2Ehrt_mul V0x) V1y)) = (ap (ap c_2Ehrt_2Ehrt_mul (ap c_2Ehrt_2Ehrt_inv V0x)) (ap c_2Ehrt_2Ehrt_inv V1y))))))$$