

thm_2Ehreal_2EHRAT_LT_ADDR
(TMUigP42uJghFoqvereZyPc2SEw6Wm6nnoS)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat \tag{3}$$

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \tag{4}$$

Definition 2 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_21$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a})))$

Definition 5 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap\ (c_2Emin_2E_40\ (ty_2Ehrat_2Ehrat\ a)))$

Let $c_2Ehrat_2Etratr_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etratr_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{c_2Ehrat_2Ehrat_REP_CLASS}) \tag{5}$$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum_2Enum)}) \quad (6)$$

Let $c_2Ehrat_2Ehrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_ABS_CLASS \in (ty_2Ehrat_2Ehrat^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})}) \quad (7)$$

Definition 6 We define $c_2Ehrat_2Ehrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 7 We define $c_2Ehrat_2Ehrat_add$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat.\lambda V1T2 \in ty_2Ehrat_2Ehrat.$

Definition 8 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ A_27a\ V0P))))$

Definition 9 We define $c_2Ehreal_2Ehrat_lt$ to be $\lambda V0x \in ty_2Ehrat_2Ehrat.\lambda V1y \in ty_2Ehrat_2Ehrat.$

Assume the following.

$$\begin{aligned} & (\forall V0h \in ty_2Ehrat_2Ehrat.(\forall V1i \in ty_2Ehrat_2Ehrat. \\ & ((ap\ (ap\ c_2Ehrat_2Ehrat_add\ V0h)\ V1i) = (ap\ (ap\ c_2Ehrat_2Ehrat_add \\ & \quad V1i)\ V0h)))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Ehrat_2Ehrat.(\forall V1y \in ty_2Ehrat_2Ehrat. \\ & (p\ (ap\ (ap\ c_2Ehreal_2Ehrat_lt\ V0x)\ (ap\ (ap\ c_2Ehrat_2Ehrat_add \\ & \quad V0x)\ V1y)))) \end{aligned} \quad (9)$$

Theorem 1

$$\begin{aligned} & (\forall V0x \in ty_2Ehrat_2Ehrat.(\forall V1y \in ty_2Ehrat_2Ehrat. \\ & (p\ (ap\ (ap\ c_2Ehreal_2Ehrat_lt\ V1y)\ (ap\ (ap\ c_2Ehrat_2Ehrat_add \\ & \quad V0x)\ V1y)))) \end{aligned}$$