

thm_2Ehreal_2EHRAT__LT__L1
(TMJX9m8CBAZkBFPUyknxq1XaNfAdQV62shC)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1Q \in 2.V1Q))\ (\lambda V2t \in 2.V2t))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{4}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{5}$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2E$

Definition 8 We define $c_2Ehrat_2Etrat_1$ to be $(ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (6)$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty ty_2Ehrat_2Ehrat \quad (7)$$

Let $c_2Ehrat_2Ehrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_ABS_CLASS \in (ty_2Ehrat_2Ehrat^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})}) \quad (8)$$

Definition 9 We define $c_2Ehrat_2Ehrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2E$

Definition 10 We define $c_2Ehrat_2Ehrat_1$ to be $(ap c_2Ehrat_2Ehrat_ABS c_2Ehrat_2Etrat_1)$.

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \quad (9)$$

Definition 11 We define $c_2Emin_2E.40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap (c_2Emin_2E.40 (ty_2$

Let $c_2Ehrat_2Etrat_inv : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_inv \in ((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (10)$$

Definition 13 We define $c_2Ehrat_2Ehrat_inv$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat.(ap c_2Ehrat_2Ehrat_ABS$

Let $c_2Ehrat_2Etrat_mul : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (11)$$

Definition 14 We define $c_2Ehrat_2Ehrat_mul$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat.\lambda V1T2 \in ty_2Ehrat_2E$

Let $c_2Ehrat_2Etrat_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (12)$$

Definition 15 We define $c_2Ehrat_2Ehrat_add$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat.\lambda V1T2 \in ty_2Ehrat_2E$

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 17 We define $c_2Ehreal_2Ehrat_lt$ to be $\lambda V0x \in ty_2Ehrat_2Ehrat. \lambda V1y \in ty_2Ehrat_2Ehrat.$

Assume the following.

$$\begin{aligned} & (\forall V0h \in ty_2Ehrat_2Ehrat. ((ap\ (ap\ c_2Ehrat_2Ehrat_mul \\ & (ap\ c_2Ehrat_2Ehrat_inv\ V0h))\ V0h) = c_2Ehrat_2Ehrat_1)) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Ehrat_2Ehrat. (\forall V1y \in ty_2Ehrat_2Ehrat. \\ & (\forall V2z \in ty_2Ehrat_2Ehrat. ((p\ (ap\ (ap\ c_2Ehreal_2Ehrat_lt \\ & (ap\ (ap\ c_2Ehrat_2Ehrat_mul\ V2z)\ V0x))\ (ap\ (ap\ c_2Ehrat_2Ehrat_mul \\ & V2z)\ V1y)))) \Leftrightarrow (p\ (ap\ (ap\ c_2Ehreal_2Ehrat_lt\ V0x)\ V1y)))))) \end{aligned} \quad (14)$$

Theorem 1

$$\begin{aligned} & (\forall V0x \in ty_2Ehrat_2Ehrat. (\forall V1y \in ty_2Ehrat_2Ehrat. \\ & ((p\ (ap\ (ap\ c_2Ehreal_2Ehrat_lt\ (ap\ (ap\ c_2Ehrat_2Ehrat_mul \\ & (ap\ c_2Ehrat_2Ehrat_inv\ V0x))\ V1y))\ c_2Ehrat_2Ehrat_1)) \Leftrightarrow (\\ & p\ (ap\ (ap\ c_2Ehreal_2Ehrat_lt\ V1y)\ V0x)))))) \end{aligned}$$