thm_2Ehreal_2EHRAT__LT__LADD (TMJnpH-weZB7S3mC1eUrmeu4GmpdX9w6N6cj)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o \ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap \ (ap \ (\text{c\_2Emin\_2E\_3D} \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \ (ap \ (\text{c\_2Emin\_2E\_3D} \ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap \ (\text{c\_2Ebool\_2E\_21} \ 2) \ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o \ (p \ P \Rightarrow p \ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap \ (ap \ \text{c\_2Emin\_2E\_3D\_3D\_3E} \ V0t) \ \text{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap \ (\text{c\_2Ebool\_2E\_21} \ 2) \ (\lambda V2t \in 2$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if} \ (\exists x \in A.p \ (ap \ P \ x)) \ \textbf{then} \ (the \ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \ V0P \ (ap \ (\text{c\_2Emin\_2E\_40} \ A$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty\_2Epair\_2Eprod$$
$$A0 \ A1)$$
$$\tag{2}$$

Let $ty\_2Ehrat\_2Ehrat : \iota$ be given. Assume the following.

$$nonempty \ ty\_2Ehrat\_2Ehrat \tag{3}$$

Let $c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehra}$$
$$\tag{4}$$

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Definition 10 *We define* c_2Ehrat_2Ehrat__REP *to be* $\lambda V0a \in ty\_2Ehrat\_2Ehrat.(ap$ (c_2Emin_2E_40 $(ty\_2$

Let $c\_2Ehrat\_2Etrat\_\_add : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty}$$
(5)

Let $c\_2Ehrat\_2Etrat\_\_eq : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2E}$$
(6)

Let $c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})})$$
(7)

Definition 11 *We define* c_2Ehrat_2Ehrat__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 12 *We define* c_2Ehrat_2Ehrat__add *to be* $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2E$

Definition 13 *We define* c_2Ehreal_2Ehrat__lt *to be* $\lambda V0x \in ty\_2Ehrat\_2Ehrat.\lambda V1y \in ty\_2Ehrat\_2Ehrat$

Assume the following.
$$True$$
(8)

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))))$$
(9)

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t))))))))$$
(10)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x))))$$
(11)

Assume the following.

$$(\forall V0h \in ty\_2Ehrat\_2Ehrat.(\forall V1i \in ty\_2Ehrat\_2Ehrat.(\forall V2j \in ty\_2Ehrat\_2Ehrat.((ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0h)\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V1i)\ V2j)) = (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0h)\ V1i))\ V2j)))))$$
(12)

Assume the following.

$$(\forall V0x \in ty\_2Ehrat\_2Ehrat.(\forall V1y \in ty\_2Ehrat\_2Ehrat.(\forall V2z \in ty\_2Ehrat\_2Ehrat.(((ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0x)\ V1y) = (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0x)\ V2z))\Leftrightarrow(V1y = V2z)))))$$
(13)

Theorem 1

$(\forall V0x \in ty\_2Ehrat\_2Ehrat.(\forall V1y \in ty\_2Ehrat\_2Ehrat.$
$(\forall V2z \in ty\_2Ehrat\_2Ehrat.((p \ (ap \ (ap \ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}$
$(ap \ (ap \ \mathsf{c\_2Ehrat\_2Ehrat\_\_add} \ V2z) \ V0x)) \ (ap \ (ap \ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}$
$V2z) \ V1y))) \Leftrightarrow (p \ (ap \ (ap \ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt} \ V0x) \ V1y))))))$