

thm\_2Ehreal\_2EHRAT\_\_LT\_\_LMUL1  
(TMd4JgWKNbNi3kJCUZw6g55jp2Dbxy94KzE)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (P \Rightarrow Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (\lambda V1Q \in 2.V1Q))\ (\lambda V2t \in 2.V2t))\ (\lambda V3t \in 2.V3t))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2))\ (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{4}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{5}$$

**Definition 7** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c\_2E$

**Definition 8** We define  $c\_2Ehrat\_2Etrat\_1$  to be  $(ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum ty\_2Enum\_2$

Let  $c\_2Ehrat\_2Etrat\_eq : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)}) \quad (6)$$

Let  $ty\_2Ehrat\_2Ehrat : \iota$  be given. Assume the following.

$$nonempty ty\_2Ehrat\_2Ehrat \quad (7)$$

Let  $c\_2Ehrat\_2Ehrat\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_ABS\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})}) \quad (8)$$

**Definition 9** We define  $c\_2Ehrat\_2Ehrat\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2E$

**Definition 10** We define  $c\_2Ehrat\_2Ehrat\_1$  to be  $(ap c\_2Ehrat\_2Ehrat\_ABS c\_2Ehrat\_2Etrat\_1)$ .

Let  $c\_2Ehrat\_2Ehrat\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehrat}) \quad (9)$$

**Definition 11** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 12** We define  $c\_2Ehrat\_2Ehrat\_REP$  to be  $\lambda V0a \in ty\_2Ehrat\_2Ehrat.(ap (c\_2Emin\_2E\_40 (ty\_2E$

Let  $c\_2Ehrat\_2Etrat\_mul : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_mul \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{ty\_2Epair\_2Eprod ty\_2Enum\_2Enum})^{ty\_2Epair\_2Eprod ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 13** We define  $c\_2Ehrat\_2Ehrat\_mul$  to be  $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2E$

Let  $c\_2Ehrat\_2Etrat\_add : \iota$  be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_add \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{ty\_2Epair\_2Eprod ty\_2Enum\_2Enum})^{ty\_2Epair\_2Eprod ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 14** We define  $c\_2Ehrat\_2Ehrat\_add$  to be  $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2E$

**Definition 15** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 16** We define  $c\_2Ehreal\_2Ehrat\_lt$  to be  $\lambda V0x \in ty\_2Ehrat\_2Ehrat.\lambda V1y \in ty\_2Ehrat\_2Ehrat$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. (\forall V1y \in A_{27a}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$(\forall V0h \in \text{ty\_2Ehtrat\_2Ehtrat}. ((\text{ap } (\text{ap } \text{c\_2Ehtrat\_2Ehtrat\_mul } \text{c\_2Ehtrat\_2Ehtrat\_1}) V0h) = V0h)) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in \text{ty\_2Ehtrat\_2Ehtrat}. (\forall V1y \in \text{ty\_2Ehtrat\_2Ehtrat}. \\ & (\forall V2z \in \text{ty\_2Ehtrat\_2Ehtrat}. ((p (\text{ap } (\text{ap } \text{c\_2Ehreal\_2Ehtrat\_lt} \\ & (\text{ap } (\text{ap } \text{c\_2Ehtrat\_2Ehtrat\_mul } V0x) V2z)) (\text{ap } (\text{ap } \text{c\_2Ehtrat\_2Ehtrat\_mul} \\ & V1y) V2z))) \Leftrightarrow (p (\text{ap } (\text{ap } \text{c\_2Ehreal\_2Ehtrat\_lt } V0x) V1y)))))) \end{aligned} \quad (14)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0x \in \text{ty\_2Ehtrat\_2Ehtrat}. (\forall V1y \in \text{ty\_2Ehtrat\_2Ehtrat}. \\ & ((p (\text{ap } (\text{ap } \text{c\_2Ehreal\_2Ehtrat\_lt } (\text{ap } (\text{ap } \text{c\_2Ehtrat\_2Ehtrat\_mul} \\ & V0x) V1y)) V1y)) \Leftrightarrow (p (\text{ap } (\text{ap } \text{c\_2Ehreal\_2Ehtrat\_lt } V0x) \text{c\_2Ehtrat\_2Ehtrat\_1})))))) \end{aligned}$$