

thm_2Ehreal_2EHRAT__LT__NE
(TMUYgwNJe2cvUi1P4EStpzY9GPNFpuCxFyf)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat \tag{3}$$

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})\ ty_2Ehrat_2Ehrat) \tag{4}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap\ P\ x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap (c_2Emin_2E_40 (ty_2Ehrat_2Ehrat_REP_CLASS\ a)))$

Let $c_2Ehrat_2Etrat_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})) \quad (5)$$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (6)$$

Let $c_2Ehrat_2Ehtrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehtrat_ABS_CLASS \in (ty_2Ehtrat_2Ehtrat^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})}) \quad (7)$$

Definition 9 We define $c_2Ehtrat_2Ehtrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 10 We define $c_2Ehtrat_2Ehtrat_add$ to be $\lambda V0T1 \in ty_2Ehtrat_2Ehtrat.\lambda V1T2 \in ty_2Ehtrat_2Ehtrat$

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 12 We define $c_2Ehreal_2Ehtrat_lt$ to be $\lambda V0x \in ty_2Ehtrat_2Ehtrat.\lambda V1y \in ty_2Ehtrat_2Ehtrat$

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (8)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (9)$$

Assume the following.

$$(\forall V0x \in ty_2Ehtrat_2Ehtrat.(\neg(p\ (ap\ (ap\ c_2Ehreal_2Ehtrat_lt\ V0x)\ V0x)))) \quad (10)$$

Theorem 1

$$(\forall V0x \in ty_2Ehtrat_2Ehtrat.(\forall V1y \in ty_2Ehtrat_2Ehtrat.((p\ (ap\ (ap\ c_2Ehreal_2Ehtrat_lt\ V0x)\ V1y)) \Rightarrow (\neg(V0x = V1y))))))$$