thm_2Ehreal_2EHRAT__LT__RADD (TMNgqMd-hWVnFExDESeTroNveneQXdnS1VtS)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Ehrat\_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehrat\_2Ehrat \tag{3}$$

Let $c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehra} \tag{4}$$

Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A{\wedge}p$ *of type* $\iota{\Rightarrow}\iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota{\Rightarrow}\iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 5 *We define* c_2Ehrat_2Ehrat__REP *to be* $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap\ ($c_2Emin_2E_40 $(ty\_2E$

Let $c\_2Ehrat\_2Etrat\_\_add : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty} \tag{5}$$

Let $c\_2Ehrat\_2Etrat\_\_eq : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \tag{6}$$

Let $c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \tag{7}$$

Definition 6 *We define* $\mathsf{c\_2Ehrat\_2Ehrat\_\_ABS}$ *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Definition 7 *We define* $\mathsf{c\_2Ehrat\_2Ehrat\_\_add}$ *to be* $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2Eh$

Definition 8 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 9 *We define* $\mathsf{c\_2Ehreal\_2Ehrat\_\_lt}$ *to be* $\lambda V0x \in ty\_2Ehrat\_2Ehrat.\lambda V1y \in ty\_2Ehrat\_2Ehrat.($

Assume the following.

$$(\forall V0h \in ty\_2Ehrat\_2Ehrat.(\forall V1i \in ty\_2Ehrat\_2Ehrat.((ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0h)\ V1i) = (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V1i)\ V0h)))) \tag{8}$$

Assume the following.

$$(\forall V0x \in ty\_2Ehrat\_2Ehrat.(\forall V1y \in ty\_2Ehrat\_2Ehrat.(\forall V2z \in ty\_2Ehrat\_2Ehrat.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V2z)\ V0x))\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V2z)\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V0x)\ V1y)))))) \tag{9}$$

Theorem 1

$$(\forall V0x \in ty\_2Ehrat\_2Ehrat.(\forall V1y \in ty\_2Ehrat\_2Ehrat.(\forall V2z \in ty\_2Ehrat\_2Ehrat.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V1y)\ V2z))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V0x)\ V1y))))))$$