thm_2Ehreal_2EHRAT__MEAN (TMVkjF3vi9LYDHL84C2ARvN8Da6AzV4BnFv)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 A

Definition 4 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 5 *We define* c_2Ebool_2ET *to be* $(ap\ (ap$ (c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ (c_2Emin_2E_3D $(2^{A\_27}$

Definition 7 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Ehrat\_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehrat\_2Ehrat \tag{3}$$

Let $c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehra} \tag{4}$$

Definition 10 *We define* c_2Ehrat_2Ehrat__REP *to be* $\lambda V0a \in ty\_2Ehrat\_2Ehrat.(ap$ (c_2Emin_2E_40 $(ty\_2$

Let $c\_2Ehrat\_2Etrat\_\_inv : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2} \quad (5)$$

Let $c\_2Ehrat\_2Etrat\_\_eq : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \quad (6)$$

Let $c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \quad (7)$$

Definition 11 *We define* c_2Ehrat_2Ehrat__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 12 *We define* c_2Ehrat_2Ehrat__inv *to be* $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.(ap$ c_2Ehrat_2Ehrat__ABS

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (8)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (9)$$

Definition 13 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (10)$$

Definition 14 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 15 *We define* c_2Ehrat_2Etrat__1 *to be* $(ap\ (ap$ (c_2Epair_2E_2C $ty\_2Enum\_2Enum\ ty\_2Enum\_$

Definition 16 *We define* c_2Ehrat_2Ehrat__1 *to be* $(ap$ c_2Ehrat_2Ehrat__ABS c_2Ehrat_2Etrat__1$)$.

Let $c\_2Ehrat\_2Etrat\_\_add : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty} \quad (11)$$

Definition 17 *We define* c_2Ehrat_2Ehrat__add *to be* $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2E$

Let $c\_2Ehrat\_2Etrat\_\_mul : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty} \quad (12)$$

Definition 18 *We define* c_2Ehrat_2Ehrat__mul *to be* $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2E$

Definition 19 *We define* c_2Ehreal_2Ehrat__lt *to be* $\lambda V0x \in ty\_2Ehrat\_2Ehrat.\lambda V1y \in ty\_2Ehrat\_2Ehrat$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{16}$$

Assume the following.

$$(\forall V0h \in ty\_2Ehrat\_2Ehrat.(\forall V1i \in ty\_2Ehrat\_2Ehrat.(\forall V2j \in ty\_2Ehrat\_2Ehrat.((ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_mul}\ V0h)\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_mul}\ V1i)\ V2j)) = (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_mul}\ V0h)\ V1i))\ V2j))))) \tag{17}$$

Assume the following.

$$(\forall V0h \in ty\_2Ehrat\_2Ehrat.(\forall V1i \in ty\_2Ehrat\_2Ehrat.(\forall V2j \in ty\_2Ehrat\_2Ehrat.((ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_mul}\ V0h)\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V1i)\ V2j)) = (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_mul}\ V0h)\ V1i))\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_mul}\ V0h)\ V2j)))))) \tag{18}$$

Assume the following.

$$(\forall V0h \in ty\_2Ehrat\_2Ehrat.((ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_mul}\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_inv}\ V0h))\ V0h) = \mathsf{c\_2Ehrat\_2Ehrat\_\_1})) \tag{19}$$

Assume the following.

$$(\forall V0x \in ty\_2Ehrat\_2Ehrat.((ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_mul}\ V0x)\ \mathsf{c\_2Ehrat\_2Ehrat\_\_1}) = V0x)) \tag{20}$$

Assume the following.

$$(\forall V0x \in ty_2Ehrat_2Ehrat.(\forall V1y \in ty_2Ehrat_2Ehrat.
(\forall V2z \in ty_2Ehrat_2Ehrat.((p\ (ap\ (ap\ \mathsf{c_2Ehreal_2Ehrat__lt}
(ap\ (ap\ \mathsf{c_2Ehrat_2Ehrat__add}\ V2z)\ V0x))\ (ap\ (ap\ \mathsf{c_2Ehrat_2Ehrat__add}
V2z)\ V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Ehreal_2Ehrat__lt}\ V0x)\ V1y)))))) \tag{21}$$

Assume the following.

$$(\forall V0x \in ty_2Ehrat_2Ehrat.(\forall V1y \in ty_2Ehrat_2Ehrat.
(\forall V2z \in ty_2Ehrat_2Ehrat.((p\ (ap\ (ap\ \mathsf{c_2Ehreal_2Ehrat__lt}
(ap\ (ap\ \mathsf{c_2Ehrat_2Ehrat__add}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c_2Ehrat_2Ehrat__add}
V1y)\ V2z)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Ehreal_2Ehrat__lt}\ V0x)\ V1y)))))) \tag{22}$$

Assume the following.

$$(\forall V0x \in ty_2Ehrat_2Ehrat.(\forall V1y \in ty_2Ehrat_2Ehrat.
(\forall V2z \in ty_2Ehrat_2Ehrat.((p\ (ap\ (ap\ \mathsf{c_2Ehreal_2Ehrat__lt}
(ap\ (ap\ \mathsf{c_2Ehrat_2Ehrat__mul}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c_2Ehrat_2Ehrat__mul}
V1y)\ V2z)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Ehreal_2Ehrat__lt}\ V0x)\ V1y)))))) \tag{23}$$

Theorem 1

$$(\forall V0x \in ty_2Ehrat_2Ehrat.(\forall V1y \in ty_2Ehrat_2Ehrat.
((p\ (ap\ (ap\ \mathsf{c_2Ehreal_2Ehrat__lt}\ V0x)\ V1y))\Rightarrow(\exists V2z \in ty_2Ehrat_2Ehrat.
((p\ (ap\ (ap\ \mathsf{c_2Ehreal_2Ehrat__lt}\ V0x)\ V2z))\wedge(p\ (ap\ (ap\ \mathsf{c_2Ehreal_2Ehrat__lt}
V2z)\ V1y)))))))$$