thm_2Ehreal_2EHREAL__LT (TMWM-rps9k19sNqnEGEvxcK6Pja86DBAL9HH)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Ehrat\_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehrat\_2Ehrat \tag{3}$$

Let $c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehra} \tag{4}$$

Definition 5 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 6 *We define* c_2Ehrat_2Ehrat__REP *to be* $\lambda V0a \in ty\_2Ehrat\_2Ehrat.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (ty\_2E$

Let $c\_2Ehrat\_2Etrat\_\_add : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty} \tag{5}$$

Let $c\_2Ehrat\_2Etrat\_\_eq : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \tag{6}$$

Let $c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \tag{7}$$

Definition 7 *We define* c_2Ehrat_2Ehrat__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Definition 8 *We define* c_2Ehrat_2Ehrat__add *to be* $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2Eh$

Definition 9 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Definition 10 *We define* c_2Ehreal_2Ehrat__lt *to be* $\lambda V0x \in ty\_2Ehrat\_2Ehrat.\lambda V1y \in ty\_2Ehrat\_2Ehrat$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{8}$$

Let $c\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$c\_2Ehreal\_2Ehreal \in (ty\_2Ehreal\_2Ehreal^{(2^{ty\_2Ehrat\_2Ehrat})}) \tag{9}$$

Let $c\_2Ehreal\_2Ecut : \iota$ be given. Assume the following.

$$c\_2Ehreal\_2Ecut \in ((2^{ty\_2Ehrat\_2Ehrat})^{ty\_2Ehreal\_2Ehreal}) \tag{10}$$

Definition 11 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 14 *We define* c_2Ehreal_2Eisacut *to be* $\lambda V0C \in (2^{ty\_2Ehrat\_2Ehrat}).(ap\ (ap$ c_2Ebool_2E_2F_5C (

Definition 15 *We define* c_2Ehreal_2Ehreal__sub *to be* $\lambda V0Y \in ty\_2Ehreal\_2Ehreal.\lambda V1X \in ty\_2Ehreal\_2$

Definition 16 *We define* c_2Ehreal_2Ehreal__add *to be* $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2$

Definition 17 *We define* c_2Ehreal_2Ehreal__lt *to be* $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2E$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg (p\ V0t)))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{15}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{17}$$

Assume the following.

$$(\forall V0h \in ty\_2Ehrat\_2Ehrat.(\forall V1i \in ty\_2Ehrat\_2Ehrat.((ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0h)\ V1i) = (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V1i)\ V0h)))) \tag{18}$$

Assume the following.

$$(\forall V0x \in ty\_2Ehrat\_2Ehrat.(\forall V1y \in ty\_2Ehrat\_2Ehrat.(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V1y)\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0x)\ V1y))))) \tag{19}$$

Assume the following.

$$(\forall V0x \in ty\_2Ehrat\_2Ehrat.(\forall V1y \in ty\_2Ehrat\_2Ehrat.(\exists V2z \in ty\_2Ehrat\_2Ehrat.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V2z)\ V0x)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V2z)\ V1y)))))) \tag{20}$$

Assume the following.

$$((\forall V0a \in ty\_2Ehreal\_2Ehreal.((ap\ c\_2Ehreal\_2Ehreal\ (ap\ c\_2Ehreal\_2Ecut\ V0a)) = V0a)) \wedge (\forall V1r \in (2^{ty\_2Ehrat\_2Ehrat}).((p\ (ap\ \mathsf{c\_2Ehreal\_2Eisacut}\ V1r)) \Leftrightarrow ((ap\ c\_2Ehreal\_2Ecut\ (ap\ c\_2Ehreal\_2Ehreal\ V1r)) = V1r)))) \tag{21}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\exists V1x \in ty\_2Ehrat\_2Ehrat.(p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V1x)))) \tag{22}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1x \in ty\_2Ehrat\_2Ehrat.(\forall V2y \in ty\_2Ehrat\_2Ehrat.(((p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V1x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V2y)\ V1x)))\Rightarrow(p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V2y)))))) \tag{23}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.(p\ (ap\ \mathsf{c\_2Ehreal\_2Eisacut}\ (\lambda V2w \in ty\_2Ehrat\_2Ehrat.(ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Ehrat\_2Ehrat)\ (\lambda V3x \in ty\_2Ehrat\_2Ehrat.(ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Ehrat\_2Ehrat)\ (\lambda V4y \in ty\_2Ehrat\_2Ehrat.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Ehrat\_2Ehrat)\ V2w)\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V3x)\ V4y)))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V3x))\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V1Y)\ V4y)))))))))))) \tag{24}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.((ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V0X)\ V1Y) = (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V1Y)\ V0X)))) \tag{25}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.(\neg((ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V0X)\ V1Y) = V0X)))) \tag{26}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V0X)\ V1Y))\Rightarrow((ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_sub}\ V1Y)\ V0X))\ V0X) = V1Y)))) \tag{27}$$

Theorem 1

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V0X)\ V1Y))\Leftrightarrow(\exists V2D \in ty\_2Ehreal\_2Ehreal.(V1Y = (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}\ V0X)\ V2D))))))$$