

thm_2Ehreal_2EHREAL__LT__TOTAL
(TMH6QnCW4jdt9dLjMfbRV3ACWkjocK9HXPC)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat \tag{3}$$

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p)$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap (c_2Emin_2E_40 (ty_2E$

Let $c_2Ehrat_2Etratr_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etratr_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \tag{5}$$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (6)$$

Let $c_2Ehrat_2Ehtrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehtrat_ABS_CLASS \in (ty_2Ehrat_2Ehtrat^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})}) \quad (7)$$

Definition 6 We define $c_2Ehrat_2Ehtrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 7 We define $c_2Ehrat_2Ehtrat_add$ to be $\lambda V0T1 \in ty_2Ehtrat_2Ehtrat.\lambda V1T2 \in ty_2Ehtrat_2Ehtrat.$

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A\ P))))$

Definition 9 We define $c_2Ehreal_2Ehtrat_lt$ to be $\lambda V0x \in ty_2Ehtrat_2Ehtrat.\lambda V1y \in ty_2Ehtrat_2Ehtrat.$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (8)$$

Let $c_2Ehreal_2Ecut : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ecut \in ((2^{ty_2Ehtrat_2Ehtrat})^{ty_2Ehreal_2Ehreal}) \quad (9)$$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_7E$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 14 We define $c_2Ehreal_2Ehreal_lt$ to be $\lambda V0X \in ty_2Ehreal_2Ehreal.\lambda V1Y \in ty_2Ehreal_2Ehreal.$

Definition 15 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ & A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(\\ & p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0X \in ty_2Ehreal_2Ehreal.(\forall V1Y \in ty_2Ehreal_2Ehreal. \\ & (((ap \ c.2Ehreal_2Ecut \ V0X) = (ap \ c.2Ehreal_2Ecut \ V1Y)) \Rightarrow (V0X = V1Y)))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0X \in ty_2Ehreal_2Ehreal.(\forall V1x \in ty_2Ehreal_2Ehreal. \\ & (\forall V2y \in ty_2Ehreal_2Ehreal.(((p \ (ap \ (ap \ c.2Ehreal_2Ecut \ V0X) \\ & V1x)) \wedge (p \ (ap \ (ap \ c.2Ehreal_2Ecut \ V0X) \ V2y) \ V1x))) \Rightarrow (p \ (ap \ (ap \ c.2Ehreal_2Ecut \\ & V0X) \ V2y)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0X \in ty_2Ehreal_2Ehreal. (\forall V1x \in ty_2Ehurat_2Ehurat. \\
& (\forall V2y \in ty_2Ehurat_2Ehurat. (((p (ap (ap c_2Ehreal_2Ecut V0X) \\
& V1x)) \wedge (\neg (p (ap (ap c_2Ehreal_2Ecut V0X) V2y)))) \Rightarrow (p (ap (ap c_2Ehreal_2Ehurat_lt \\
& V1x) V2y))))))
\end{aligned} \tag{22}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{23}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{26}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))))) \quad (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q)) \vee \neg(p \ V0p)))))) \quad (32)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0X \in ty_2Ehreal_2Ehreal. (\forall V1Y \in ty_2Ehreal_2Ehreal. \\
& ((V0X = V1Y) \vee ((p \ (ap \ (ap \ c_2Ehreal_2Ehreal_lt \ V0X) \ V1Y)) \vee (p \ (ap \\
& (ap \ c_2Ehreal_2Ehreal_lt \ V1Y) \ V0X))))))
\end{aligned}$$