

thm_2Ehreal_2EHREAL__MUL__ISACUT
 (TMRj9NFXKAXTd1XxHCXfM3TQCvEUKtHKrsh)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^\omega) \quad (3)$$

Definition 5 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V1t2 = V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod \\ & \quad A0\ A1) \end{aligned} \quad (4)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ & \quad A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (5)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum) (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum))$

Definition 9 We define $c_2Ehrat_2Etrat_1$ to be $(ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)))$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})$$
(6)

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat$$
(7)

Let $c_2Ehrat_2Ehrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_ABS_CLASS \in (ty_2Ehrat_2Ehrat)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})}$$
(8)

Definition 10 We define $c_2Ehrat_2Ehrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum) (ty_2Ehrat_2Ehrat)$

Definition 11 We define $c_2Ehrat_2Ehrat_1$ to be $(ap c_2Ehrat_2Ehrat_ABS c_2Ehrat_2Etrat_1)$.

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat})$$
(9)

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge P x) \text{ of type } \iota \Rightarrow \iota)$.

Definition 13 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat. (ap (c_2Emin_2E_40 (ty_2Ehrat_2Ehrat)))$

Let $c_2Ehrat_2Etrat_inv : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_inv \in ((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum))^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}$$
(10)

Definition 14 We define $c_2Ehrat_2Ehrat_inv$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat. (ap c_2Ehrat_2Ehrat_ABS (c_2Ehrat_2Etrat_inv))$

Let $c_2Ehrat_2Etrat_mul : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum))^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})$$
(11)

Definition 15 We define $c_2Ehrat_2Ehrat_mul$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat. \lambda V1T2 \in ty_2Ehrat_2Ehrat. (ap (c_2Ehrat_2Etrat_mul (c_2Ehrat_2Etrat_inv (ty_2Ehrat_2Ehrat))) (ty_2Ehrat_2Ehrat))$

Let $c_2Ehrat_2Etrat_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum))^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})$$
(12)

Definition 16 We define $c_2Ehrat_2Ehrat_add$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat. \lambda V1T2 \in ty_2Ehrat_2Ehrat. (ap (c_2Ehrat_2Etrat_add (c_2Ehrat_2Etrat_inv (ty_2Ehrat_2Ehrat))) (ty_2Ehrat_2Ehrat))$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 18 We define $c_2Ehreal_2Ehrat_lt$ to be $\lambda V0x \in ty_2Ehrat_2Ehrat. \lambda V1y \in ty_2Ehrat_2Ehrat$

Definition 19 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 20 We define $c_2Ehreal_2Eisacut$ to be $\lambda V0C \in (2^{ty_2Ehrat_2Ehrat}).(ap\ (ap\ c_2Ebool_2E_2F_5C\ ($

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (13)$$

Let $c_2Ehreal_2Ecute : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ecute \in ((2^{ty_2Ehrat_2Ehrat})^{ty_2Ehreal_2Ehreal}) \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (20)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((\text{True} \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow \text{True}) \Leftrightarrow (p V0t)) \wedge (((\text{False} \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(p V0t))))))) \quad (23)$$

Assume the following.

$$(\forall V0h \in \text{ty_2Ehrat_2Ehrat}. (\forall V1i \in \text{ty_2Ehrat_2Ehrat}. ((ap (ap c_2Ehrat_2Ehrat_mul V0h) V1i) = (ap (ap c_2Ehrat_2Ehrat_mul V1i) V0h)))) \quad (24)$$

Assume the following.

$$(\forall V0h \in \text{ty_2Ehrat_2Ehrat}. (\forall V1i \in \text{ty_2Ehrat_2Ehrat}. (\forall V2j \in \text{ty_2Ehrat_2Ehrat}. ((ap (ap c_2Ehrat_2Ehrat_mul V0h) (ap (ap c_2Ehrat_2Ehrat_mul V1i) V2j)) = (ap (ap c_2Ehrat_2Ehrat_mul (ap (ap c_2Ehrat_2Ehrat_mul V0h) V1i)) V2j))))))) \quad (25)$$

Assume the following.

$$(\forall V0h \in \text{ty_2Ehrat_2Ehrat}. ((ap (ap c_2Ehrat_2Ehrat_mul c_2Ehrat_2Ehrat_1) V0h) = V0h)) \quad (26)$$

Assume the following.

$$(\forall V0h \in \text{ty_2Ehrat_2Ehrat}. ((ap (ap c_2Ehrat_2Ehrat_mul (ap c_2Ehrat_2Ehrat_inv V0h)) V0h) = c_2Ehrat_2Ehrat_1)) \quad (27)$$

Assume the following.

$$(\forall V0x \in \text{ty_2Ehrat_2Ehrat}. (\forall V1y \in \text{ty_2Ehrat_2Ehrat}. ((p (ap (ap c_2Ehrat_2Ehrat_lt V0x) V1y)) \Rightarrow (\neg(V0x = V1y))))) \quad (28)$$

Assume the following.

$$(\forall V0u \in \text{ty_2Ehrat_2Ehrat}. (\forall V1v \in \text{ty_2Ehrat_2Ehrat}. (\forall V2x \in \text{ty_2Ehrat_2Ehrat}. (\forall V3y \in \text{ty_2Ehrat_2Ehrat}. (((p (ap (ap c_2Ehrat_2Ehrat_lt V0u) V2x)) \wedge (p (ap (ap c_2Ehrat_2Ehrat_lt V1v) V3y))) \Rightarrow (p (ap (ap c_2Ehrat_2Ehrat_lt (ap (ap c_2Ehrat_2Ehrat_mul V0u) V1v)) (ap (ap c_2Ehrat_2Ehrat_mul V2x) V3y))))))))))) \quad (29)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Ehrat_2Ehrat. (\forall V1y \in ty_2Ehrat_2Ehrat. \\
 & (\forall V2z \in ty_2Ehrat_2Ehrat. ((p (ap (ap c_2Ehreal_2Ehrat_lt \\
 & (ap (ap c_2Ehrat_2Ehrat_mul V0x) V2z)) (ap (ap c_2Ehrat_2Ehrat_mul \\
 & V1y) V2z))) \Leftrightarrow (p (ap (ap c_2Ehreal_2Ehrat_lt V0x) V1y)))))))
 \end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Ehrat_2Ehrat. (\forall V1y \in ty_2Ehrat_2Ehrat. \\
 & ((p (ap (ap c_2Ehreal_2Ehrat_lt (ap (ap c_2Ehrat_2Ehrat_mul \\
 & V0x) V1y)) V1y)) \Leftrightarrow (p (ap (ap c_2Ehreal_2Ehrat_lt V0x) c_2Ehrat_2Ehrat_1))))))
 \end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Ehrat_2Ehrat. (\forall V1y \in ty_2Ehrat_2Ehrat. \\
 & ((p (ap (ap c_2Ehreal_2Ehrat_lt (ap (ap c_2Ehrat_2Ehrat_mul \\
 & V0x) (ap c_2Ehrat_2Ehrat_inv V1y))) c_2Ehrat_2Ehrat_1)) \Leftrightarrow \\
 & p (ap (ap c_2Ehreal_2Ehrat_lt V0x) V1y))))
 \end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0X \in ty_2Ehreal_2Ehreal. (\exists V1x \in ty_2Ehrat_2Ehrat. \\
 & (p (ap (ap c_2Ehreal_2Ecut V0X) V1x)))
 \end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0X \in ty_2Ehreal_2Ehreal. (\exists V1x \in ty_2Ehrat_2Ehrat. \\
 & (\neg(p (ap (ap c_2Ehreal_2Ecut V0X) V1x))))
 \end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0X \in ty_2Ehreal_2Ehreal. (\forall V1x \in ty_2Ehrat_2Ehrat. \\
 & (\forall V2y \in ty_2Ehrat_2Ehrat. (((p (ap (ap c_2Ehreal_2Ecut V0X) \\
 & V1x)) \wedge (p (ap (ap c_2Ehreal_2Ehrat_lt V2y) V1x))) \Rightarrow (p (ap (ap c_2Ehreal_2Ecut \\
 & V0X) V2y))))))
 \end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0X \in ty_2Ehreal_2Ehreal. (\forall V1x \in ty_2Ehrat_2Ehrat. \\
 & ((p (ap (ap c_2Ehreal_2Ecut V0X) V1x)) \Rightarrow (\exists V2y \in ty_2Ehrat_2Ehrat. \\
 & ((p (ap (ap c_2Ehreal_2Ecut V0X) V2y)) \wedge (p (ap (ap c_2Ehreal_2Ehrat_lt \\
 & V1x) V2y)))))))
 \end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0X \in ty_2Ehreal_2Ehreal. (\forall V1x \in ty_2Ehrat_2Ehrat. \\
 & (\forall V2y \in ty_2Ehrat_2Ehrat. (((p (ap (ap c_2Ehreal_2Ecut V0X) \\
 & V1x)) \wedge (\neg(p (ap (ap c_2Ehreal_2Ecut V0X) V2y)))) \Rightarrow (p (ap (ap c_2Ehreal_2Ehrat_lt \\
 & V1x) V2y))))))
 \end{aligned} \tag{37}$$

Theorem 1

$$\begin{aligned} & (\forall V0X \in ty_2Ehreal_2Ehreal. (\forall V1Y \in ty_2Ehreal_2Ehreal. \\ & (p (ap c_2Ehreal_2Eisacut (\lambda V2w \in ty_2Ehrat_2Ehrat. (ap (c_2Ebool_2E_3F \\ & ty_2Ehrat_2Ehrat) (\lambda V3x \in ty_2Ehrat_2Ehrat. (ap (c_2Ebool_2E_3F \\ & ty_2Ehrat_2Ehrat) (\lambda V4y \in ty_2Ehrat_2Ehrat. (ap (ap c_2Ebool_2E_2F_5C \\ & (ap (ap (c_2Emin_2E_3D ty_2Ehrat_2Ehrat) V2w) (ap (ap c_2Ehrat_2Ehrat_mul \\ & V3x) V4y))) (ap (ap c_2Ebool_2E_2F_5C (ap (ap c_2Ehreal_2Ecut V0X) \\ & V3x)) (ap (ap c_2Ehreal_2Ecut V1Y) V4y))))))))))) \end{aligned}$$