thm_2Ehreal_2EHREAL__SUB__ADD (TMZHuQbz-TYG5mQFaqBnjwjdTuSvL2bR6Uyp)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Ehrat\_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehrat\_2Ehrat \tag{3}$$

Let $c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehra} \tag{4}$$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ehrat_2Ehrat__REP *to be* $\lambda V0a \in ty\_2Ehrat\_2Ehrat.(ap$ (c_2Emin_2E_40 $(ty\_2E$

Let $c\_2Ehrat\_2Etrat\_\_add : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty} \tag{5}$$

Let $c\_2Ehrat\_2Etrat\_\_eq : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \tag{6}$$

Let $c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \tag{7}$$

Definition 9 *We define* c_2Ehrat_2Ehrat__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Definition 10 *We define* c_2Ehrat_2Ehrat__add *to be* $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2E$

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 12 *We define* c_2Ehreal_2Ehrat__lt *to be* $\lambda V0x \in ty\_2Ehrat\_2Ehrat.\lambda V1y \in ty\_2Ehrat\_2Ehrat$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{8}$$

Let $c\_2Ehreal\_2Ecut : \iota$ be given. Assume the following.

$$c\_2Ehreal\_2Ecut \in ((2^{ty\_2Ehrat\_2Ehrat})^{ty\_2Ehreal\_2Ehreal}) \tag{9}$$

Definition 13 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$c\_2Ehreal\_2Ehreal \in (ty\_2Ehreal\_2Ehreal^{(2^{ty\_2Ehrat\_2Ehrat})}) \tag{10}$$

Definition 14 *We define* c_2Ehreal_2Ehreal__add *to be* $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2$

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 16 *We define* c_2Ehreal_2Ehreal__sub *to be* $\lambda V0Y \in ty\_2Ehreal\_2Ehreal.\lambda V1X \in ty\_2Ehreal\_2$

Definition 17 *We define* c_2Ehreal_2Eisacut *to be* $\lambda V0C \in (2^{ty\_2Ehrat\_2Ehrat}).(ap\ (ap$ c_2Ebool_2E_2F_5C (

Definition 18 *We define* c_2Ehreal_2Ehreal__lt *to be* $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2E$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\lor(\neg(p\ V0t)))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(((True\land(p\ V0t))\Leftrightarrow(p\ V0t))\land((((p\ V0t)\land True)\Leftrightarrow(p\ V0t))\land(((False\land(p\ V0t))\Leftrightarrow False)\land((((p\ V0t)\land False)\Leftrightarrow False)\land(((p\ V0t)\land(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{15}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\land(((\neg True)\Leftrightarrow False)\land((\neg False)\Leftrightarrow True))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\land((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\land(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\land(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{18}$$

Assume the following.

$$(\forall V0h \in ty\_2Ehrat\_2Ehrat.(\forall V1i \in ty\_2Ehrat\_2Ehrat.((ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0h)\ V1i) = (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V1i)\ V0h)))) \tag{19}$$

Assume the following.

$$(\forall V0h \in ty\_2Ehrat\_2Ehrat.(\forall V1i \in ty\_2Ehrat\_2Ehrat.(\forall V2j \in ty\_2Ehrat\_2Ehrat.((ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0h)\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V1i)\ V2j)) = (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0h)\ V1i))\ V2j))))) \tag{20}$$

Assume the following.

$$(\forall V0x \in ty\_2Ehrat\_2Ehrat.(\forall V1y \in ty\_2Ehrat\_2Ehrat. (p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V1y)\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0x)\ V1y))))) \tag{21}$$

Assume the following.

$$(\forall V0x \in ty\_2Ehrat\_2Ehrat.(\forall V1y \in ty\_2Ehrat\_2Ehrat. (\forall V2z \in ty\_2Ehrat\_2Ehrat.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V2z)\ V0x))\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V2z)\ V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V0x)\ V1y)))))) \tag{22}$$

Assume the following.

$$(\forall V0x \in ty\_2Ehrat\_2Ehrat.(\forall V1y \in ty\_2Ehrat\_2Ehrat. (\forall V2z \in ty\_2Ehrat\_2Ehrat.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V1y)\ V2z)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V0x)\ V1y)))))) \tag{23}$$

Assume the following.

$$(\forall V0x \in ty\_2Ehrat\_2Ehrat.(\forall V1y \in ty\_2Ehrat\_2Ehrat. (\exists V2z \in ty\_2Ehrat\_2Ehrat.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V2z)\ V0x))\land(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V2z)\ V1y)))))) \tag{24}$$

Assume the following.

$$((\forall V0a \in ty\_2Ehreal\_2Ehreal.((ap\ c\_2Ehreal\_2Ehreal\ (ap\ c\_2Ehreal\_2Ecut\ V0a)) = V0a))\land(\forall V1r \in (2^{ty\_2Ehrat\_2Ehrat}). ((p\ (ap\ \mathsf{c\_2Ehreal\_2Eisacut}\ V1r))\Leftrightarrow((ap\ c\_2Ehreal\_2Ecut\ (ap\ c\_2Ehreal\_2Ehreal\ V1r)) = V1r)))) \tag{25}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal. (((ap\ c\_2Ehreal\_2Ecut\ V0X) = (ap\ c\_2Ehreal\_2Ecut\ V1Y))\Rightarrow(V0X = V1Y)))) \tag{26}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1x \in ty\_2Ehrat\_2Ehrat. (\forall V2y \in ty\_2Ehrat\_2Ehrat.(((p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V1x))\land(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V2y)\ V1x)))\Rightarrow(p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V2y)))))) \tag{27}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1x \in ty\_2Ehrat\_2Ehrat. ((p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V1x))\Rightarrow(\exists V2y \in ty\_2Ehrat\_2Ehrat. ((p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V2y))\land(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V1x)\ V2y))))))) \tag{28}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1x \in ty\_2Ehrat\_2Ehrat.(\forall V2y \in ty\_2Ehrat\_2Ehrat.(((\neg(p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V1x)))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V1x)\ V2y)))\Rightarrow(\neg(p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V2y)))))))) \tag{29}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1x \in ty\_2Ehrat\_2Ehrat.(\forall V2y \in ty\_2Ehrat\_2Ehrat.(((p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V1x))\wedge(\neg(p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V2y))))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehrat\_\_lt}\ V1x)\ V2y)))))) \tag{30}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1e \in ty\_2Ehrat\_2Ehrat.(\exists V2x \in ty\_2Ehrat\_2Ehrat.((p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V2x))\wedge(\neg(p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V2x)\ V1e)))))))) \tag{31}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.(p\ (ap\ \mathsf{c\_2Ehreal\_2Eisacut}\ (\lambda V2w \in ty\_2Ehrat\_2Ehrat.(ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Ehrat\_2Ehrat)\ (\lambda V3x \in ty\_2Ehrat\_2Ehrat.(ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Ehrat\_2Ehrat)\ (\lambda V4y \in ty\_2Ehrat\_2Ehrat.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Ehrat\_2Ehrat)\ V2w)\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V3x)\ V4y)))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V3x))\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V1Y)\ V4y)))))))))))) \tag{32}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V0X)\ V1Y))\Rightarrow(\exists V2x \in ty\_2Ehrat\_2Ehrat.((\neg(p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V2x)))\wedge(p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V1Y)\ V2x)))))))) \tag{33}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V0X)\ V1Y))\Rightarrow(p\ (ap\ \mathsf{c\_2Ehreal\_2Eisacut}\ (\lambda V2w \in ty\_2Ehrat\_2Ehrat.(ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Ehrat\_2Ehrat)\ (\lambda V3x \in ty\_2Ehrat\_2Ehrat.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V3x)))\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V1Y)\ (ap\ (ap\ \mathsf{c\_2Ehrat\_2Ehrat\_\_add}\ V3x)\ V2w))))))))))) \tag{34}$$

Theorem 1

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V0X)\ V1Y)) \Rightarrow ((ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_add}$$
$$(ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_sub}\ V1Y)\ V0X))\ V0X) = V1Y))))$$