thm_2Ehreal_2EHREAL__SUP (TMR-fRMyK41bemeLbnKXvDfQayP9WVhYLXV8)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let $ty\_2Ehrat\_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehrat\_2Ehrat \tag{2}$$

Let $c\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$c\_2Ehreal\_2Ehreal \in (ty\_2Ehreal\_2Ehreal^{(2^{ty\_2Ehrat\_2Ehrat})}) \tag{3}$$

Let $c\_2Ehreal\_2Ecut : \iota$ be given. Assume the following.

$$c\_2Ehreal\_2Ecut \in ((2^{ty\_2Ehrat\_2Ehrat})^{ty\_2Ehreal\_2Ehreal}) \tag{4}$$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (\mathsf{c_2Emin_2E_40}\ $

Definition 9 *We define* c_2Ehreal_2Ehreal__sup *to be* $\lambda V0P \in (2^{ty\_2Ehreal\_2Ehreal}).(ap\ c\_2Ehreal\_2Ehreal$

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 11 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{5}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{6}$$

Let $c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Ehrat\_2Ehra} \tag{7}$$

Definition 12 *We define* c_2Ehrat_2Ehrat__REP *to be* $\lambda V0a \in ty\_2Ehrat\_2Ehrat.(ap$ (c_2Emin_2E_40 $(ty\_2$

Let $c\_2Ehrat\_2Etrat\_\_add : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty} \tag{8}$$

Let $c\_2Ehrat\_2Etrat\_\_eq : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Etrat\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \tag{9}$$

Let $c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Ehrat\_2Ehrat\_\_ABS\_\_CLASS \in (ty\_2Ehrat\_2Ehrat^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \tag{10}$$

Definition 13 *We define* c_2Ehrat_2Ehrat__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 14 *We define* c_2Ehrat_2Ehrat__add *to be* $\lambda V0T1 \in ty\_2Ehrat\_2Ehrat.\lambda V1T2 \in ty\_2Ehrat\_2E$

Definition 15 *We define* c_2Ehreal_2Ehrat__lt *to be* $\lambda V0x \in ty\_2Ehrat\_2Ehrat.\lambda V1y \in ty\_2Ehrat\_2Ehrat$

Definition 16 *We define* c_2Ehreal_2Eisacut *to be* $\lambda V0C \in (2^{ty\_2Ehrat\_2Ehrat}).(ap\ (ap$ c_2Ebool_2E_2F_5C (

Definition 17 *We define* c_2Ehreal_2Ehreal__lt *to be* $\lambda V0X \in ty\_2Ehreal\_2Ehreal.\lambda V1Y \in ty\_2Ehreal\_2E$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{15}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{18}$$

Assume the following.

$$((\forall V0a \in ty\_2Ehreal\_2Ehreal.((ap\ c\_2Ehreal\_2Ehreal\ (ap\ c\_2Ehreal\_2Ecut\ V0a)) = V0a)) \wedge (\forall V1r \in (2^{ty\_2Ehrat\_2Ehrat}).((p\ (ap\ \mathsf{c\_2Ehreal\_2Eisacut}\ V1r)) \Leftrightarrow ((ap\ c\_2Ehreal\_2Ecut\ (ap\ c\_2Ehreal\_2Ehreal\ V1r)) = V1r)))) \tag{19}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V0X)\ V1Y)) \Rightarrow (\exists V2x \in ty\_2Ehrat\_2Ehrat.((\neg (p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V0X)\ V2x))) \wedge (p\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V1Y)\ V2x))))))) \tag{20}$$

Assume the following.

$$(\forall V0X \in ty\_2Ehreal\_2Ehreal.(\forall V1Y \in ty\_2Ehreal\_2Ehreal.((V0X = V1Y) \vee ((p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V0X)\ V1Y)) \vee (p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V1Y)\ V0X)))))) \tag{21}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Ehreal\_2Ehreal}).(((\exists V1X \in ty\_2Ehreal\_2Ehreal.(p\ (ap\ V0P\ V1X)))\wedge(\exists V2Y \in ty\_2Ehreal\_2Ehreal.(\forall V3X \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ V0P\ V3X))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V3X)\ V2Y))))))\Rightarrow(p\ (ap\ \mathsf{c\_2Ehreal\_2Eisacut}\ (\lambda V4w \in ty\_2Ehrat\_2Ehrat.(ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Ehreal\_2Ehreal)\ (\lambda V5X \in ty\_2Ehreal\_2Ehreal.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ V0P\ V5X))\ (ap\ (ap\ c\_2Ehreal\_2Ecut\ V5X)\ V4w)))))))))) \tag{22}$$

Theorem 1

$$(\forall V0P \in (2^{ty\_2Ehreal\_2Ehreal}).(((\exists V1X \in ty\_2Ehreal\_2Ehreal.(p\ (ap\ V0P\ V1X)))\wedge(\exists V2Y \in ty\_2Ehreal\_2Ehreal.(\forall V3X \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ V0P\ V3X))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V3X)\ V2Y))))))\Rightarrow(\forall V4Y \in ty\_2Ehreal\_2Ehreal.((\exists V5X \in ty\_2Ehreal\_2Ehreal.((p\ (ap\ V0P\ V5X))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V4Y)\ V5X))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_lt}\ V4Y)\ (ap\ \mathsf{c\_2Ehreal\_2Ehreal\_\_sup}\ V0P)))))))$$