

thm_2Ehreal_2EHREAL_SUP_ISACUT (TMSgSG19ghYp6HspDL6NfWJMff9xU8nzGbc)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat \tag{3}$$

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap (c_2Emin_2E_40 (ty_2E$

Let $c_2Ehrat_2Etratr_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etratr_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \tag{5}$$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (6)$$

Let $c_2Ehrat_2Ehtrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehtrat_ABS_CLASS \in (ty_2Ehrat_2Ehtrat)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (7)$$

Definition 6 We define $c_2Ehrat_2Ehtrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 7 We define $c_2Ehrat_2Ehtrat_add$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehtrat.\lambda V1T2 \in ty_2Ehrat_2Ehtrat.$

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a\ P))))$

Definition 9 We define $c_2Ehreal_2Ehtrat_lt$ to be $\lambda V0x \in ty_2Ehrat_2Ehtrat.\lambda V1y \in ty_2Ehrat_2Ehtrat.$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.(ap\ (c_2Emin_2E_3D_3D_3E\ V0t1)\ V2t))))$

Definition 12 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_2F_5C))$

Definition 14 We define $c_2Ehreal_2Eisacut$ to be $\lambda V0C \in (2^{ty_2Ehtrat_2Ehtrat}).(ap\ (ap\ c_2Ebool_2E_2F_5C\ C))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (8)$$

Let $c_2Ehreal_2Ecut : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ecut \in ((2^{ty_2Ehtrat_2Ehtrat})^{ty_2Ehreal_2Ehreal}) \quad (9)$$

Definition 15 We define $c_2Ehreal_2Ehreal_lt$ to be $\lambda V0X \in ty_2Ehreal_2Ehreal.\lambda V1Y \in ty_2Ehreal_2Ehreal.$

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.(ap\ (c_2Emin_2E_3D_3D_3E\ V0t1)\ V2t))))$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (13)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (14)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t)))))) \quad (15)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0X \in ty_2Ehreal_2Ehreal.(\exists V1x \in ty_2Ehrat_2Ehrat. \\
& (p \ (ap \ (ap \ c_2Ehreal_2Ecut \ V0X) \ V1x)))) \quad (16)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0X \in ty_2Ehreal_2Ehreal.(\exists V1x \in ty_2Ehrat_2Ehrat. \\
& (\neg(p \ (ap \ (ap \ c_2Ehreal_2Ecut \ V0X) \ V1x)))) \quad (17)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0X \in ty_2Ehreal_2Ehreal.(\forall V1x \in ty_2Ehrat_2Ehrat. \\
& (\forall V2y \in ty_2Ehrat_2Ehrat.(((p \ (ap \ (ap \ c_2Ehreal_2Ecut \ V0X) \\
& V1x)) \wedge (p \ (ap \ (ap \ c_2Ehreal_2Ehrat_lt \ V2y) \ V1x))) \Rightarrow (p \ (ap \ (ap \ c_2Ehreal_2Ecut \\
& V0X) \ V2y)))))) \quad (18)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0X \in ty_2Ehreal_2Ehreal.(\forall V1x \in ty_2Ehrat_2Ehrat. \\
& ((p \ (ap \ (ap \ c_2Ehreal_2Ecut \ V0X) \ V1x)) \Rightarrow (\exists V2y \in ty_2Ehrat_2Ehrat. \\
& ((p \ (ap \ (ap \ c_2Ehreal_2Ecut \ V0X) \ V2y)) \wedge (p \ (ap \ (ap \ c_2Ehreal_2Ehrat_lt \\
& V1x) \ V2y)))))) \quad (19)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow False))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (24)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (25)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (26)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (28)$$

Theorem 1

$$(\forall V0P \in (2^{ty_2Ehreal_2Ehreal}). (((\exists V1X \in ty_2Ehreal_2Ehreal. (p (ap V0P V1X))) \wedge (\exists V2Y \in ty_2Ehreal_2Ehreal. (\forall V3X \in ty_2Ehreal_2Ehreal. ((p (ap V0P V3X)) \Rightarrow (p (ap (ap c_2Ehreal_2Ehreal_lt V3X) V2Y)))))) \Rightarrow (p (ap c_2Ehreal_2Eisacut (\lambda V4w \in ty_2Ehreal_2Ehreal. (ap (c_2Ebool_2E_3F ty_2Ehreal_2Ehreal) (\lambda V5X \in ty_2Ehreal_2Ehreal. (ap (ap c_2Ebool_2E_2F_5C (ap V0P V5X)) (ap (ap c_2Ehreal_2Ecut V5X) V4w)))))))))) \quad (29)$$