

thm_2Ehreal_2EISACUT__HRAT (TMdg- wnh5LueYfuUQVmKWRo982fPsWFDjhAS)

October 26, 2020

Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x))$

Definition 3 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V 0P \in (2^{A_27a}). (ap V 0P (ap (c_2Emin_2E_40 A_27a) P)))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat \tag{3}$$

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \tag{4}$$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V 0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}) P) (c_2Emin_2E_40 A_27a) P)))$

Definition 6 We define `c_2Ehrat_2Ehrat__REP` to be $\lambda V 0a \in ty_2Ehrat_2Ehrat. (ap (c_2Emin_2E_40 (ty_2Ehrat_2Ehrat_REP_CLASS V 0a) (ty_2Ehrat_2Ehrat_REP_CLASS V 0a)))$

Let $c_2Ehrat_2Etratl_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etratl_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \tag{5}$$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (6)$$

Let $c_2Ehrat_2Ehrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_ABS_CLASS \in (ty_2Ehrat_2Ehrat^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})}) \quad (7)$$

Definition 7 We define $c_2Ehrat_2Ehrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 8 We define $c_2Ehrat_2Ehrat_add$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat.\lambda V1T2 \in ty_2Ehrat_2Ehrat.$

Definition 9 We define $c_2Ehreal_2Ehrat_lt$ to be $\lambda V0x \in ty_2Ehrat_2Ehrat.\lambda V1y \in ty_2Ehrat_2Ehrat.$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.))$

Definition 12 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_2F_5C\ V0t))$

Definition 14 We define $c_2Ehreal_2Eisacut$ to be $\lambda V0C \in (2^{ty_2Ehrat_2Ehrat}).(ap\ (ap\ c_2Ebool_2E_2F_5C\ V0C\ V0t))$

Definition 15 We define $c_2Ehreal_2Ecut_of_hrat$ to be $\lambda V0x \in ty_2Ehrat_2Ehrat.(\lambda V1y \in ty_2Ehrat_2Ehrat.$

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.))$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \wedge (p\ V1t2)) \Leftrightarrow ((p\ V1t2) \wedge (p\ V0t1)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (11)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Ehrrat_2Ehrrat. (\forall V1y \in ty_2Ehrrat_2Ehrrat. \\
& (\forall V2z \in ty_2Ehrrat_2Ehrrat. (((p (ap (ap c_2Ehrral_2Ehrrat_lt \\
& V0x) V1y)) \wedge (p (ap (ap c_2Ehrral_2Ehrrat_lt V1y) V2z))) \Rightarrow (p (ap (\\
& ap c_2Ehrral_2Ehrrat_lt V0x) V2z))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Ehrrat_2Ehrrat. (\forall V1y \in ty_2Ehrrat_2Ehrrat. \\
& (\neg((p (ap (ap c_2Ehrral_2Ehrrat_lt V0x) V1y)) \wedge (p (ap (ap c_2Ehrral_2Ehrrat_lt \\
& V1y) V0x))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Ehrrat_2Ehrrat. (\exists V1y \in ty_2Ehrrat_2Ehrrat. \\
& (p (ap (ap c_2Ehrral_2Ehrrat_lt V0x) V1y))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Ehrrat_2Ehrrat. (\exists V1y \in ty_2Ehrrat_2Ehrrat. \\
& (p (ap (ap c_2Ehrral_2Ehrrat_lt V1y) V0x))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Ehrrat_2Ehrrat. (\forall V1y \in ty_2Ehrrat_2Ehrrat. \\
& ((p (ap (ap c_2Ehrral_2Ehrrat_lt V0x) V1y)) \Rightarrow (\exists V2z \in ty_2Ehrrat_2Ehrrat. \\
& ((p (ap (ap c_2Ehrral_2Ehrrat_lt V0x) V2z)) \wedge (p (ap (ap c_2Ehrral_2Ehrrat_lt \\
& V2z) V1y))))))
\end{aligned} \tag{16}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{17}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{20}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \wedge (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee ((\neg(p \vee 1q)) \vee (\neg(p \vee 2r)))) \wedge (((p \vee 1q) \vee \\
& (\neg(p \vee 0p))) \wedge ((p \vee 2r) \vee (\neg(p \vee 0p))))))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee 0p) \Leftrightarrow (\neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee \\
& (p \vee 1q)) \wedge ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p))))))
\end{aligned} \tag{23}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (p \vee 0p)))) \tag{24}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (\neg(p \vee 1q)))) \tag{25}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \vee 0p))) \Rightarrow (p \vee 0p))) \tag{26}$$

Theorem 1

$$\begin{aligned}
& (\forall V0h \in ty_2Ehrrat_2Ehrrat. (p \ (ap \ c_2Ehreal_2Eisacut \ (ap \\
& \ c_2Ehreal_2Ecut_of_hrrat \ V0h))))
\end{aligned}$$