

thm_2Eieeee_2Enum2roundmode__11
(TMWtTaNf9Eyiugk1foHQjAV1TnTSJ2peajP)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p \Rightarrow Q)$ of type ι .

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Enum_2E0 to be $(ap \ c_2Enum_2EABS_num \ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 5 We define c_2Ebool_2ET to be $(ap \ (ap \ (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap \ (ap \ (c_2Emin_2E_3D \ (2^{A-27a})))$

Definition 7 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap \ c_2Enum_2EABS_num \ (ap \ c_2Enum_2ESUC_REP \ m))$

Let $c_2Earithmic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 8 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2E_2B$

Definition 9 We define $c_2Earithmic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2E_2B$

Definition 10 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2))$

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Definition 14 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 16 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Eieeee_2Eroundmode : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eieeee_2Eroundmode \quad (7)$$

Let $c_2Eieeee_2Eroundmode2num : \iota$ be given. Assume the following.

$$c_2Eieeee_2Eroundmode2num \in (ty_2Enum_2Enum^{ty_2Eieeee_2Eroundmode}) \quad (8)$$

Let $c_2Eieeee_2Enum2roundmode : \iota$ be given. Assume the following.

$$c_2Eieeee_2Enum2roundmode \in (ty_2Eieeee_2Eroundmode^{ty_2Enum_2Enum}) \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (10)$$

Assume the following.

$$\begin{aligned} & ((\forall V0a \in ty_2Eieeee_2Eroundmode.((ap c_2Eieeee_2Enum2roundmode \\ & (ap c_2Eieeee_2Eroundmode2num V0a) = V0a) \wedge (\forall V1r \in ty_2Enum_2Enum. \\ & ((p (ap (\lambda V2n \in ty_2Enum_2Enum.(ap (ap c_2Eprim_rec_2E_3C \\ & V2n) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 \\ & (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) V1r) \Leftrightarrow \\ & ((ap c_2Eieeee_2Eroundmode2num (ap c_2Eieeee_2Enum2roundmode \\ & V1r) = V1r)))))) \end{aligned} \quad (11)$$

Theorem 1

$$\begin{aligned} & (\forall V0r \in ty_2Enum_2Enum. (\forall V1r_27 \in ty_2Enum_2Enum. \\ & ((p (ap (ap c_2Eprim_rec_2E_3C V0r) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Rightarrow \\ & ((p (ap (ap c_2Eprim_rec_2E_3C V1r_27) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Rightarrow \\ & (((ap c_2Eieee_2Enum2roundmode V0r) = (ap c_2Eieee_2Enum2roundmode \\ & V1r_27)) \Leftrightarrow (V0r = V1r_27)))))) \end{aligned}$$