

thm_2Eieeee_2Eroundmode2num__11
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 pZuq8oBtMca5HQjEofzGhaBJ4z)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 5 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda \tau a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a})))$

Definition 7 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 8 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B$

Definition 9 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 11 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2))$

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V2t \in 2.V2t)))$

Definition 14 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^A)^{27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ V0P)))$

Definition 16 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V1n$

Let $ty_2Eieeee_2Eroundmode : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eieeee_2Eroundmode \quad (7)$$

Let $c_2Eieeee_2Eroundmode2num : \iota$ be given. Assume the following.

$$c_2Eieeee_2Eroundmode2num \in (ty_2Enum_2Enum^{ty_2Eieeee_2Eroundmode}) \quad (8)$$

Let $c_2Eieeee_2Enum2roundmode : \iota$ be given. Assume the following.

$$c_2Eieeee_2Enum2roundmode \in (ty_2Eieeee_2Eroundmode^{ty_2Enum_2Enum}) \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (10)$$

Assume the following.

$$\begin{aligned} & ((\forall V0a \in ty_2Eieeee_2Eroundmode.((ap\ c_2Eieeee_2Enum2roundmode \\ & (ap\ c_2Eieeee_2Eroundmode2num\ V0a)) = V0a)) \wedge (\forall V1r \in ty_2Enum_2Enum. \\ & ((p\ (ap\ (\lambda V2n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Eprim_rec_2E_3C \\ & V2n)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2 \\ & (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ V1r)) \Leftrightarrow \\ & ((ap\ c_2Eieeee_2Eroundmode2num\ (ap\ c_2Eieeee_2Enum2roundmode \\ & V1r)) = V1r)))) \end{aligned} \quad (11)$$

Theorem 1

$(\forall V0a \in ty_2Eieee_2Eroundmode. (\forall V1a_27 \in ty_2Eieee_2Eroundmode. ((ap_c_2Eieee_2Eroundmode2num\ V0a) = (ap_c_2Eieee_2Eroundmode2num\ V1a_27)) \Leftrightarrow (V0a = V1a_27))))$