

thm_2Eieeee_2Eroundmode2num__ONTO
(TMZqV-
cuN4LGdd3r96ZVYcYj5VDvHC3hsBMh)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A \cdot 27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40 } A$

Definition 4 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow p \ Q)$ of type ι .

Let `c_2Enum_2EZERO__REP` : ι be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{2}$$

Let `c_2Enum_2EABS__num` : ι be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 5 We define `c_2Enum_2E0` to be $(\text{ap } c_2Enum_2EABS_num \ c_2Enum_2EZERO_REP).$

Definition 6 We define `c_2Earithmic_2EZERO` to be `c_2Enum_2E0`.

Let `c_2Enum_2EREP__num` : ι be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let `c_2Enum_2ESUC__REP` : ι be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 7 We define c_Ebool_2ET to be $(ap (ap (c_Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 8 We define $c_Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_Emin_2E_3D (2^{A_27a}))$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 10 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B$

Definition 11 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B$

Definition 12 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 13 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 14 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_7E$

Definition 15 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_Ebool_2E_21 2) (\lambda V2t \in 2. V2t$

Definition 16 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. V1n$

Let $ty_2Eieee_2Eroundmode : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eieee_2Eroundmode \quad (7)$$

Let $c_2Eieee_2Eroundmode2num : \iota$ be given. Assume the following.

$$c_2Eieee_2Eroundmode2num \in (ty_2Enum_2Enum^{ty_2Eieee_2Eroundmode}) \quad (8)$$

Let $c_2Eieee_2Enum2roundmode : \iota$ be given. Assume the following.

$$c_2Eieee_2Enum2roundmode \in (ty_2Eieee_2Eroundmode^{ty_2Enum_2Enum}) \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (10)$$

Assume the following.

$$\begin{aligned} & ((\forall V0a \in ty_2Eieee_2Eroundmode. ((ap\ c_2Eieee_2Enum2roundmode \\ & (ap\ c_2Eieee_2Eroundmode2num\ V0a)) = V0a)) \wedge (\forall V1r \in ty_2Enum_2Enum. \\ & ((p\ (ap\ (\lambda V2n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Eprim_rec_2E_3C \\ & V2n)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2 \\ & (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ V1r)) \Leftrightarrow \\ & ((ap\ c_2Eieee_2Eroundmode2num\ (ap\ c_2Eieee_2Enum2roundmode\ V1r)) = V1r)))) \end{aligned} \quad (11)$$

Theorem 1

$$\begin{aligned} & (\forall V0r \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\ & V0r) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Leftrightarrow (\exists V1a \in \\ & ty_2Eieee_2Eroundmode. (V0r = (ap c_2Eieee_2Eroundmode2num V1a)))))) \end{aligned}$$