

thm_2Eieee_2Erroundmode__case__def (TMLtPCXLXKaxjtYcpgYfaFr7pJ4EHPSzCfY)

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Let $ty_2Eieee_2Erroundmode : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eieee_2Erroundmode \quad (1)$$

Let $c_2Eieee_2ETo_ninfinity : \iota$ be given. Assume the following.

$$c_2Eieee_2ETo_ninfinity \in ty_2Eieee_2Erroundmode \quad (2)$$

Let $c_2Eieee_2ETo_pinfinity : \iota$ be given. Assume the following.

$$c_2Eieee_2ETo_pinfinity \in ty_2Eieee_2Erroundmode \quad (3)$$

Let $c_2Eieee_2Efloat_To_zero : \iota$ be given. Assume the following.

$$c_2Eieee_2Efloat_To_zero \in ty_2Eieee_2Erroundmode \quad (4)$$

Let $c_2Eieee_2ETo_nearest : \iota$ be given. Assume the following.

$$c_2Eieee_2ETo_nearest \in ty_2Eieee_2Erroundmode \quad (5)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (6)$$

Let $c_2Eieee_2Erroundmode2num : \iota$ be given. Assume the following.

$$c_2Eieee_2Erroundmode2num \in (ty_2Enum_2Enum^{ty_2Eieee_2Erroundmode}) \quad (7)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p \text{ of type } \iota \Rightarrow \iota)$.

Definition 8 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (9)$$

Definition 9 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 10 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (11)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 12 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E$

Definition 13 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 16 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 17 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E$

Definition 18 We define $c_2Eieee_2Eroundmode_CASE$ to be $\lambda A_27a : \iota.\lambda V0x \in ty_2Eieee_2Eroundmode.$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (14)$$

Definition 19 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap (c_2Earithmetic_2EODD m) n)$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (V2t)))$

Definition 21 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap (c_2Earithmetic_2E_3E m) n)$

Definition 22 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap (c_2Earithmetic_2E_3E_3D m) n)$

Definition 23 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2E_5C_2F m) (c_2Earithmetic_2E_3C_3D (V1t1))) (c_2Earithmetic_2E_3C_3D (V1t2))))$

Let $c_2Earithmetic_2EEEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (17)$$

Definition 24 We define $c_2Eenumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (18)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A_27a. nonempty A_27a \Rightarrow & (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Eieee_2Erroundmode2num c_2Eieee_2ETO_nearest) = c_2Enum_2E0) \wedge \\
& \quad (((ap c_2Eieee_2Erroundmode2num c_2Eieee_2Efloat_To_zero) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \wedge \\
& \quad (((ap c_2Eieee_2Erroundmode2num c_2Eieee_2ETO_pinfinity) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((ap c_2Eieee_2Erroundmode2num c_2Eieee_2ETO_ninfinity) = (\\
& \quad ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO)))))))
\end{aligned} \tag{21}$$

Assume the following.

$((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2EiZ (ap (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum.((\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A V6n) V7m))))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V10n) V11m))))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \wedge ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2EEEXP V15n) V16m))))))) \wedge (((ap c_2Enum_2ESUC c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge ((\forall V17n \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2ESUC V17n))))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Eprim_rec_2EPRE V18n))))))) \wedge ((\forall V19n \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum.((\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m))))))) \wedge ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V23n)) \Leftrightarrow False))) \wedge ((\forall V24n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V24n))))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.((\forall V26m \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V25n)) (ap c_2Earithmetic_2ENUMERAL V26m)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V25n) V26m))))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V27n)) \Leftrightarrow False))) \wedge ((\forall V28n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL V28n)) c_2Enum_2E0) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V28n))))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.((\forall V30m \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V29n)) (ap c_2Earithmetic_2ENUMERAL V30m)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V29n))))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V31n)) \Leftrightarrow True))) \wedge ((\forall V32n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V32n)) \Leftrightarrow False))) \wedge ((\forall V33n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V33n)) \Leftrightarrow True))) \wedge ((\forall V34n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V34n)) \Leftrightarrow False)))$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. ((c_2Earithmetic_2EZERO = (ap c_2Earithmetic_2EBIT1 V0n)) \Leftrightarrow False) \wedge \\
& (((ap c_2Earithmetic_2EBIT1 V0n) = c_2Earithmetic_2EZERO) \Leftrightarrow \\
& False) \wedge (((c_2Earithmetic_2EZERO = (ap c_2Earithmetic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap c_2Earithmetic_2EBIT2 V0n) = c_2Earithmetic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap c_2Earithmetic_2EBIT1 V0n) = (ap c_2Earithmetic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c_2Earithmetic_2EBIT2 V0n) = (ap c_2Earithmetic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c_2Earithmetic_2EBIT1 V0n) = (ap c_2Earithmetic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c_2Earithmetic_2EBIT2 V0n) = (ap c_2Earithmetic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))) \\
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 \\
& V0n)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& (ap c_2Earithmetic_2EBIT2 V0n)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmetic_2EZERO) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT1 V1m)) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m)) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT2 V1m)) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m)) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT2 V1m)) \Leftrightarrow \\
& (\neg(p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT1 V1m)) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))) \\
\end{aligned} \tag{24}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow ((\forall V0v0 \in A_27a. (\forall V1v1 \in \\
& A_27a. (\forall V2v2 \in A_27a. (\forall V3v3 \in A_27a. ((ap (ap (ap \\
& ap (ap (c_2Eieee_2Erroundmode_CASE A_27a) c_2Eieee_2ETo_nearest \\
& V0v0) V1v1) V2v2) V3v3) = V0v0)))))) \wedge ((\forall V4v0 \in A_27a. (\forall V5v1 \in \\
& A_27a. (\forall V6v2 \in A_27a. (\forall V7v3 \in A_27a. ((ap (ap (ap \\
& ap (ap (c_2Eieee_2Erroundmode_CASE A_27a) c_2Eieee_2Efloat_To_zero) \\
& V4v0) V5v1) V6v2) V7v3) = V5v1)))))) \wedge ((\forall V8v0 \in A_27a. (\forall V9v1 \in \\
& A_27a. (\forall V10v2 \in A_27a. (\forall V11v3 \in A_27a. ((ap (ap (ap \\
& (ap (ap (c_2Eieee_2Erroundmode_CASE A_27a) c_2Eieee_2ETo_pinfinity) \\
& V8v0) V9v1) V10v2) V11v3) = V10v2)))))) \wedge ((\forall V12v0 \in A_27a. \\
& \forall V13v1 \in A_27a. (\forall V14v2 \in A_27a. (\forall V15v3 \in A_27a. \\
& ((ap (ap (ap (ap (ap (c_2Eieee_2Erroundmode_CASE A_27a) c_2Eieee_2ETo_ninfinity) \\
& V12v0) V13v1) V14v2) V15v3) = V15v3))))))) \\
\end{aligned}$$