thm_2Eind__type_2ECONSTR__IND (TMVpVyeg22n9QXfBnSPtjoyVF41cCvZRdhG)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota{\Rightarrow}\iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota{\Rightarrow}\iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Let $c\_2Eind\_\_type\_2ENUMSND : \iota$ be given. Assume the following.

$$c\_2Eind\_\_type\_2ENUMSND \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Eind\_\_type\_2ENUMFST : \iota$ be given. Assume the following.

$$c\_2Eind\_\_type\_2ENUMFST \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{3}$$

Definition 6 *We define* c_2Eind__type_2EINJF *to be* $\lambda A\_27a : \iota.\lambda V0f \in (((2^{A\_27a})^{ty\_2Enum\_2Enum})^{ty\_2Enum}$

Definition 7 *We define* c_2Eind__type_2EINJA *to be* $\lambda A\_27a : \iota.\lambda V0a \in A\_27a.(\lambda V1n \in ty\_2Enum\_2Enum$

Let $c\_2Eind\_\_type\_2ENUMRIGHT : \iota$ be given. Assume the following.

$$c\_2Eind\_\_type\_2ENUMRIGHT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Eind\_\_type\_2ENUMLEFT : \iota$ be given. Assume the following.

$$c\_2Eind\_\_type\_2ENUMLEFT \in (2^{ty\_2Enum\_2Enum}) \tag{5}$$

Definition 8 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 11 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 12 *We define* c_2Eind__type_2EINJP *to be* $\lambda A\_27a : \iota.\lambda V0f1 \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).\lambda V1f$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{6}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{7}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{8}$$

Definition 13 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 14 *We define* c_2Eind__type_2EINJN *to be* $\lambda A\_27a : \iota.\lambda V0m \in ty\_2Enum\_2Enum.(\lambda V1n \in ty\_2$

Definition 15 *We define* c_2Eind__type_2EZCONSTR *to be* $\lambda A\_27a : \iota.\lambda V0c \in ty\_2Enum\_2Enum.\lambda V1i \in$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{9}$$

Definition 16 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 17 *We define* c_2Eind__type_2EZBOT *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Eind\_\_type\_2EINJP}\ A\_27a)\ ($

Definition 18 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 19 *We define* c_2Eind__type_2EZRECSPACE *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}$

Let $ty\_2Eind\_\_type\_2Erecspace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eind\_\_type\_2Erecspace\ A0) \tag{10}$$

Let $c\_2Eind\_\_type\_2Emk\_\_rec : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eind\_\_type\_2Emk\_\_rec\ A\_27a \in ((ty\_2Eind\_\_type\_2Erecspace\ A\_27a)^{((2^{A\_27a})^{ty\_2Enum\_2Enum})}) \tag{11}$$

Definition 20 *We define* c_2Eind__type_2EBOTTOM *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eind\_\_type\_2Emk\_\_rec\ A\_27$

Let $c\_2Eind\_\_type\_2Edest\_\_rec : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eind\_\_type\_2Edest\_\_rec\ A\_27a \in (((2^{A\_27a})^{ty\_2Enum\_2Enum})^{(ty\_2Eind\_\_type\_2Erecspace\ A\_27a)}) \quad (12)$$

Definition 21 *We define* c_2Eind__type_2ECONSTR *to be* $\lambda A\_27a : \iota.\lambda V0c \in ty\_2Enum\_2Enum.\lambda V1i \in A$

Definition 22 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \quad (17)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{22}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{23}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w)))))))) \tag{24}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow ((p\ V1y) \vee (p\ V3w)))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in (ty\_2Eind\_\_type\_2Erecspace\ A\_27a).((ap\ (c\_2Eind\_\_type\_2Emk\_\_rec\ A\_27a)\ (ap\ (c\_2Eind\_\_type\_2Edest\_\_rec\ A\_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).((p\ (ap\ (\mathsf{c\_2Eind\_\_type\_2EZRECSPACE}\ A\_27a)\ V1r)) \Leftrightarrow ((ap\ (c\_2Eind\_\_type\_2Edest\_\_rec\ A\_27a)\ (ap\ (c\_2Eind\_\_type\_2Emk\_\_rec\ A\_27a)\ V1r)) = V1r)))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{30}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{31}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{32}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{33}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{34}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{35}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{36}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{37}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{(ty\_2Eind\_\_type\_2Erecspace\ \ A\_27a)}). (((p\ (ap\ V0P\ (\mathsf{c\_2Eind\_\_type\_2EBOTTOM}\ A\_27a)))\wedge(\forall V1c \in ty\_2Enum\_2Enum. (\forall V2i \in A\_27a.(\forall V3r \in ((ty\_2Eind\_\_type\_2Erecspace\ A\_27a)^{ty\_2Enum\_2Enum}).((\forall V4n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ (ap\ V3r\ V4n))))\Rightarrow(p\ (ap\ V0P\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eind\_\_type\_2ECONSTR}\ A\_27a)\ V1c)\ V2i)\ V3r))))))))\Rightarrow(\forall V5x \in (ty\_2Eind\_\_type\_2Erecspace\ A\_27a).(p\ (ap\ V0P\ V5x)))))$$