thm_2Eind__type_2ECONSTR__REC (TMdedgYTqDquXaS8wuHmfZ4F1k4UevhkxMf)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\text{c\_2Emin\_2E\_40}\ A$

Definition 10 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ \text{c\_2Ebool\_2E\_2F\_5C}$

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2E}$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Eind\_\_type\_2Erecspace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eind\_\_type\_2Erecspace\ A0) \tag{2}$$

Let $c\_2Eind\_\_type\_2Edest\_\_rec : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eind\_\_type\_2Edest\_\_rec\ A\_27a \in (((2^{A\_27a})^{ty\_2Enum\_2Enum})^{(ty\_2Eind\_\_type\_2Erecspace\ A\_27a)}) \tag{3}$$

Let $c\_2Eind\_\_type\_2ENUMSND : \iota$ be given. Assume the following.

$$c\_2Eind\_\_type\_2ENUMSND \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (4)$$

Let $c\_2Eind\_\_type\_2ENUMFST : \iota$ be given. Assume the following.

$$c\_2Eind\_\_type\_2ENUMFST \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (5)$$

Definition 12 *We define* c_2Eind__type_2EINJF *to be* $\lambda A\_27a : \iota.\lambda V0f \in (((2^{A\_27a})^{ty\_2Enum\_2Enum})^{ty\_2Enu}$

Definition 13 *We define* c_2Eind__type_2EINJA *to be* $\lambda A\_27a : \iota.\lambda V0a \in A\_27a.(\lambda V1n \in ty\_2Enum\_2Enu$

Let $c\_2Eind\_\_type\_2ENUMRIGHT : \iota$ be given. Assume the following.

$$c\_2Eind\_\_type\_2ENUMRIGHT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (6)$$

Let $c\_2Eind\_\_type\_2ENUMLEFT : \iota$ be given. Assume the following.

$$c\_2Eind\_\_type\_2ENUMLEFT \in (2^{ty\_2Enum\_2Enum}) \quad (7)$$

Definition 14 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 15 *We define* c_2Eind__type_2EINJP *to be* $\lambda A\_27a : \iota.\lambda V0f1 \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).\lambda V1f$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (8)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (9)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (10)$$

Definition 16 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 17 *We define* c_2Eind__type_2EINJN *to be* $\lambda A\_27a : \iota.\lambda V0m \in ty\_2Enum\_2Enum.(\lambda V1n \in ty\_2$

Definition 18 *We define* c_2Eind__type_2EZCONSTR *to be* $\lambda A\_27a : \iota.\lambda V0c \in ty\_2Enum\_2Enum.\lambda V1i \in$

Let $c\_2Eind\_\_type\_2Emk\_\_rec : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eind\_\_type\_2Emk\_\_rec\ A\_27a \in ((ty\_2Eind\_\_type\_2Erecspace\ A\_27a)^{((2^{A\_27a})^{ty\_2Enum\_2Enum})}) \quad (11)$$

Definition 19 *We define* c_2Eind__type_2ECONSTR *to be* $\lambda A\_27a : \iota.\lambda V0c \in ty\_2Enum\_2Enum.\lambda V1i \in A$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{12}$$

Definition 20 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 21 *We define* c_2Eind__type_2EZBOT *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Eind\_\_type\_2EINJP}\ A\_27a)\ (a$

Definition 22 *We define* c_2Eind__type_2EBOTTOM *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eind\_\_type\_2Emk\_\_rec\ A\_27$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{17}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{26}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F\_21}\ A\_27a)\ (\lambda V1x \in A\_27a.(ap\ V0P\ V1x)))) \Leftrightarrow ((\exists V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \wedge (\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(((p\ (ap\ V0P\ V3x)) \wedge (p\ (ap\ V0P\ V4y))) \Rightarrow (V3x = V4y)))))))) \tag{28}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{29}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w))))))))\tag{30}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow ((p\ V1y) \vee (p\ V3w))))))))\tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x)))))))\tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A\_27a.(p\ (ap\ V1Q\ V4x)))))))\tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in A\_27a.(\exists V1x \in A\_27a.(V1x = V0a)))\tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in A\_27a.(p\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F\_21}\ A\_27a)\ (\lambda V1x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V1x)\ V0a)))))\tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))))\tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in A\_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b^{A\_27a}).(\forall V4x \in A\_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))))\tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1t \in A\_27a.((\forall V2x \in A\_27a.((V2x = V1t) \Rightarrow (p\ (ap\ V0P\ V2x)))) \Rightarrow (p\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ A\_27a)\ V0P))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in ty\_2Enum\_2Enum.(\forall V1i \in A\_27a.(\forall V2r \in ((ty\_2Eind\_\_type\_2Erecspace\ A\_27a)^{ty\_2Enum\_2Enum}).(\neg((ap\ (ap\ (ap\ (\mathsf{c\_2Eind\_\_type\_2ECONSTR}\ A\_27a)\ V0c)\ V1i)\ V2r) = (\mathsf{c\_2Eind\_\_type\_2EBOTTOM}\ A\_27a)))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c1 \in ty\_2Enum\_2Enum.(\forall V1i1 \in A\_27a.(\forall V2r1 \in ((ty\_2Eind\_\_type\_2Erecspace\ A\_27a)^{ty\_2Enum\_2Enum}).(\forall V3c2 \in ty\_2Enum\_2Enum.(\forall V4i2 \in A\_27a.(\forall V5r2 \in ((ty\_2Eind\_\_type\_2Erecspace\ A\_27a)^{ty\_2Enum\_2Enum}).(((ap\ (ap\ (ap\ (\mathsf{c\_2Eind\_\_type\_2ECONSTR}\ A\_27a)\ V0c1)\ V1i1)\ V2r1) = (ap\ (ap\ (ap\ (\mathsf{c\_2Eind\_\_type\_2ECONSTR}\ A\_27a)\ V3c2)\ V4i2)\ V5r2)) \Leftrightarrow ((V0c1 = V3c2) \wedge ((V1i1 = V4i2) \wedge (V2r1 = V5r2)))))))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Eind\_\_type\_2Erecspace\ A\_27a)}).(((p\ (ap\ V0P\ (\mathsf{c\_2Eind\_\_type\_2EBOTTOM}\ A\_27a))) \wedge (\forall V1c \in ty\_2Enum\_2Enum.(\forall V2i \in A\_27a.(\forall V3r \in ((ty\_2Eind\_\_type\_2Erecspace\ A\_27a)^{ty\_2Enum\_2Enum}).((\forall V4n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ (ap\ V3r\ V4n)))) \Rightarrow (p\ (ap\ V0P\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eind\_\_type\_2ECONSTR}\ A\_27a)\ V1c)\ V2i)\ V3r)))))))) \Rightarrow (\forall V5x \in (ty\_2Eind\_\_type\_2Erecspace\ A\_27a).(p\ (ap\ V0P\ V5x))))) \tag{41}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0Fn \in ((((A\_27b^{(A\_27b^{ty\_2Enum\_2Enum})})^{((ty\_2Eind\_\_type\_2Erecspace\ A\_27a)^{ty\_2Enum\_2Enum})})^{A\_27a})^{ty\_2Enum\_2}$$

$$(\exists V1f \in (A\_27b^{(ty\_2Eind\_\_type\_2Erecspace\ A\_27a)}).(\forall V2c \in ty\_2Enum\_2Enum.(\forall V3i \in A\_27a.(\forall V4r \in ((ty\_2Eind\_\_type\_2Erecspace\ A\_27a)^{ty\_2Enum\_2Enum}).((ap\ V1f\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eind\_\_type\_2ECONSTR}\ A\_27a)\ V2c)\ V3i)\ V4r)) = (ap\ (ap\ (ap\ (ap\ V0Fn\ V2c)\ V3i)\ V4r)\ (\lambda V5n \in ty\_2Enum\_2Enum.(ap\ V1f\ (ap\ V4r\ V5n))))))))))$$