

# thm\_2Eind\_\_type\_2EINJ\_\_INVERSE2 (TMJd- BeVWRTZLaVsByfWiF6HxNd3Bxjwe4KG)

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**Definition 1** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2E_T` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A-27a}))))$

**Definition 5** We define `c_2Ebool_2E_F` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V0t \in 2. V0t))$ .

**Definition 6** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (the (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V2t \in 2. V2t))$

**Definition 8** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P (\text{ap } (\text{c\_2Emin\_2E\_40 } A))))$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2. (((\text{True} \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge \text{True}) \Leftrightarrow (p V0t)) \wedge (((\text{False} \wedge (p V0t)) \Leftrightarrow \text{False}) \wedge (((p V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{3}$$

Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow (\forall V0x \in A. 27a. ((V0x = V0x) \Leftrightarrow \text{True})) \tag{4}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. ((ap (c.2Emin.2E.40 A.27a) (\lambda V1y \in A.27a. (ap (ap (c.2Emin.2E.3D A.27a) V1y) V0x))) = (5) \\ V0x))$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1a \in A.27a. ((\exists V2x \in A.27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (6) \\ ap V0P V1a))))))$$

**Theorem 1**

$$\forall A.27A.nonempty A.27A \Rightarrow \forall A.27B.nonempty A.27B \Rightarrow \forall A.27C. \\ nonempty A.27C \Rightarrow (\forall V0P \in ((A.27C^{A.27B})^{A.27A}). ((\forall V1x1 \in A.27A. (\forall V2y1 \in A.27B. (\forall V3x2 \in A.27A. (\forall V4y2 \in A.27B. (((ap (ap V0P V1x1) V2y1) = (ap (ap V0P V3x2) V4y2)) \Leftrightarrow ((V1x1 = V3x2) \wedge (V2y1 = V4y2)))))) \Rightarrow (\exists V5X \in (A.27A^{A.27C}). (\exists V6Y \in (A.27B^{A.27C}). (\forall V7x \in A.27A. (\forall V8y \in A.27B. (((ap V5X (ap (ap V0P V7x) V8y)) = V7x) \wedge ((ap V6Y (ap (ap V0P V7x) V8y)) = V8y))))))))))$$