thm_2Eind__type_2EZCONSTR__ZBOT (TMe12ayAq6BNbRVdDVBhyx2tpbjQb5NAkKr)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Eind\_\_type\_2ENUMSND : \iota$ be given. Assume the following.

$$c\_2Eind\_\_type\_2ENUMSND \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Eind\_\_type\_2ENUMFST : \iota$ be given. Assume the following.

$$c\_2Eind\_\_type\_2ENUMFST \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{3}$$

Definition 7 *We define* c_2Eind__type_2EINJF *to be* $\lambda A\_27a : \iota.\lambda V0f \in (((2^{A\_27a})^{ty\_2Enum\_2Enum})^{ty\_2Enum}$

Definition 8 *We define* c_2Eind__type_2EINJA *to be* $\lambda A\_27a : \iota.\lambda V0a \in A\_27a.(\lambda V1n \in ty\_2Enum\_2Enum$

Let $c\_2Eind\_\_type\_2ENUMRIGHT : \iota$ be given. Assume the following.

$$c\_2Eind\_\_type\_2ENUMRIGHT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Eind\_\_type\_2ENUMLEFT : \iota$ be given. Assume the following.

$$c\_2Eind\_\_type\_2ENUMLEFT \in (2^{ty\_2Enum\_2Enum}) \tag{5}$$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 11 *We define* c_2Eind__type_2EINJP *to be* $\lambda A\_27a : \iota.\lambda V0f1 \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).\lambda V1f$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{6}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{7}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{8}$$

Definition 12 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 13 *We define* c_2Eind__type_2EINJN *to be* $\lambda A\_27a : \iota.\lambda V0m \in ty\_2Enum\_2Enum.(\lambda V1n \in ty\_2$

Definition 14 *We define* c_2Eind__type_2EZCONSTR *to be* $\lambda A\_27a : \iota.\lambda V0c \in ty\_2Enum\_2Enum.\lambda V1i \in$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{9}$$

Definition 15 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 16 *We define* c_2Eind__type_2EZBOT *to be* $\lambda A_27a : \iota.(ap\ (ap\ ($c_2Eind__type_2EINJP $A\_27a)\ (a$

Definition 17 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Assume the following.

$$True \tag{10}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{14}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n1 \in ty\_2Enum\_2Enum.(\forall V1n2 \in ty\_2Enum\_2Enum.(((ap\ (\mathsf{c\_2Eind\_\_type\_2EINJN}\ A\_27a)\ V0n1) = (ap\ (\mathsf{c\_2Eind\_\_type\_2EINJN}\ A\_27a)\ V1n2)) \Leftrightarrow (V0n1 = V1n2)))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f1 \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).(\forall V1f1\_27 \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).(\forall V2f2 \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).(\forall V3f2\_27 \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).(((ap\ (ap\ (\mathsf{c\_2Eind\_\_type\_2EINJP}\ A\_27a)\ V0f1)\ V2f2) = (ap\ (ap\ (\mathsf{c\_2Eind\_\_type\_2EINJP}\ A\_27a)\ V1f1\_27)\ V3f2\_27)) \Leftrightarrow ((V0f1 = V1f1\_27) \wedge (V2f2 = V3f2\_27))))))) \tag{17}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = \mathsf{c\_2Enum\_2E0}))) \tag{18}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in ty\_2Enum\_2Enum.(\forall V1i \in A\_27a.(\forall V2r \in (((2^{A\_27a})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}).(\neg((ap\ (ap\ (ap\ (\mathsf{c\_2Eind\_\_type\_2EZCONSTR}\ A\_27a)\ V0c)\ V1i)\ V2r) = (\mathsf{c\_2Eind\_\_type\_2EZBOT}\ A\_27a))))))$$