

thm_2Eind__type_2EZRECSpace__ind
(TMQGjk-
VARy7xRE4xWFN2rs3paH75gCB7zvH)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Eind_type_2ENUMSND : \iota$ be given. Assume the following.

$$c_2Eind_type_2ENUMSND \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Eind_type_2ENUMFST : \iota$ be given. Assume the following.

$$c_2Eind_type_2ENUMFST \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \tag{3}$$

Definition 7 We define $c_2Eind_type_2EINJF$ to be $\lambda A_27a : \iota.\lambda V0f \in (((2^{A_27a})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})$

Definition 8 We define $c_2Eind_type_2EINJA$ to be $\lambda A_27a : \iota.\lambda V0a \in A_27a.(\lambda V1n \in ty_2Enum_2Enum)$

Let $c_2Eind_type_2ENUMRIGHT : \iota$ be given. Assume the following.

$$c_2Eind_type_2ENUMRIGHT \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Eind_type_2ENUMLEFT : \iota$ be given. Assume the following.

$$c_2Eind_type_2ENUMLEFT \in (2^{ty_2Enum_2Enum}) \tag{5}$$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$).

Definition 11 We define c_2Ebool_2ECOND to be $\lambda A.\lambda a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda V2t2 \in A.\lambda V3t3 \in A.$

Definition 12 We define $c_2Eind_type_2EINJP$ to be $\lambda A.\lambda a : \iota.\lambda V0f1 \in ((2^{A.\lambda a})^{ty_2Enum_2Enum}).\lambda V1f2 \in$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 13 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 14 We define $c_2Eind_type_2EINJN$ to be $\lambda A.\lambda a : \iota.\lambda V0m \in ty_2Enum_2Enum.(\lambda V1n \in ty_2Enum_2Enum.$

Definition 15 We define $c_2Eind_type_2EZCONSTR$ to be $\lambda A.\lambda a : \iota.\lambda V0c \in ty_2Enum_2Enum.\lambda V1i \in 2^A.$

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A.\lambda a}).(ap V0P (ap (c_2Emin_2E_40$

Let $c_2Enum_2EZZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZZERO_REP \in \omega \quad (9)$$

Definition 17 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZZERO_REP)$.

Definition 18 We define $c_2Eind_type_2EZBOT$ to be $\lambda A.\lambda a : \iota.(ap (ap (c_2Eind_type_2EINJP A.\lambda a) (ap$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 20 We define $c_2Eind_type_2EZRECSpace$ to be $\lambda A.\lambda a : \iota.(\lambda V0a0 \in ((2^{A.\lambda a})^{ty_2Enum_2Enum}$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in \\ & 2.((((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w))) \Rightarrow (((p V0x) \wedge (p V2z)) \Rightarrow \\ & ((p V1y) \wedge (p V3w)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in \\ & 2.((((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w))) \Rightarrow (((p V0x) \vee (p V2z)) \Rightarrow \\ & ((p V1y) \vee (p V3w)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \Rightarrow (p (ap V1Q V2x)))) \Rightarrow \\ & ((\forall V3x \in A.27a.(p (ap V0P V3x))) \Rightarrow (\forall V4x \in A.27a.(p (\\ & ap V1Q V4x)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \Rightarrow (p (ap V1Q V2x)))) \Rightarrow \\ & ((\exists V3x \in A.27a.(p (ap V0P V3x))) \Rightarrow (\exists V4x \in A.27a.(p (\\ & ap V1Q V4x)))))) \end{aligned} \quad (16)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0ZRECSpace_27 \in (2^{((2^{A.27a})^{ty_2Enum_2Enum})}). \\ & (((p (ap V0ZRECSpace_27 (c.2Eind_type_2EZBOT A.27a))) \wedge (\forall V1c \in \\ & ty_2Enum_2Enum.(\forall V2i \in A.27a.(\forall V3r \in (((2^{A.27a})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}). \\ & ((\forall V4n \in ty_2Enum_2Enum.(p (ap V0ZRECSpace_27 (ap V3r V4n)))) \Rightarrow \\ & (p (ap V0ZRECSpace_27 (ap (ap (c.2Eind_type_2EZCONSTR A.27a) \\ & V1c) V2i) V3r)))))) \Rightarrow (\forall V5a0 \in ((2^{A.27a})^{ty_2Enum_2Enum}). \\ & ((p (ap (c.2Eind_type_2EZRECSpace A.27a) V5a0)) \Rightarrow (p (ap V0ZRECSpace_27 \\ & V5a0)))))) \end{aligned}$$