thm_2EindexedLists_2EEL__MAP2i (TMWBF-MumkAjrYiowojU8jkKDocFNktYi2Xt)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Definition 4 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 5 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 9 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2EindexedLists\_2EMAP2i : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow c\_2EindexedLists\_2EMAP2i\ A\_27a\ A\_27b\ A\_27c \in ((((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27c)})^{(ty\_2Elist\_2Elist\ A\_27b)})^{(((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum})}) \tag{3}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{4}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{5}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{6}$$

Let $c\_2Elist\_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EHD\ A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{7}$$

Let $c\_2Elist\_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEL\ A\_27a \in ((A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{10}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{11}$$

Definition 12 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{12}$$

Definition 13 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 14 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = \mathsf{c\_2Enum\_2E0})\vee(\exists V1n \in ty\_2Enum\_2Enum.(V0m = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))))) \tag{13}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))))) \tag{14}$$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow(p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge(((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{22}$$

Assume the following.

$$
\begin{gathered}
(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1){\Rightarrow} \\
((p\ V1t2){\Rightarrow}(p\ V2t3))){\Leftrightarrow}(((p\ V0t1){\wedge}(p\ V1t2)){\Rightarrow}(p\ V2t3))))))
\end{gathered} \tag{23}
$$

Assume the following.

$$
\begin{gathered}
(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\
2.((((p\ V0x){\Leftrightarrow}(p\ V1x\_27)){\wedge}((p\ V1x\_27){\Rightarrow}((p\ V2y){\Leftrightarrow}(p\ V3y\_27)))){\Rightarrow} \\
(((p\ V0x){\Rightarrow}(p\ V2y)){\Leftrightarrow}((p\ V1x\_27){\Rightarrow}(p\ V3y\_27))))))))
\end{gathered} \tag{24}
$$

Assume the following.

$$
\begin{gathered}
\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}\forall A\_27c. \\
nonempty\ A\_27c{\Rightarrow}(\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}). \\
(\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a) \\
V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))
\end{gathered} \tag{25}
$$

Assume the following.

$$
\begin{gathered}
\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}\forall A\_27c. \\
nonempty\ A\_27c{\Rightarrow}(\forall V0P \in (((2^{(ty\_2Elist\_2Elist\ A\_27c)})^{(ty\_2Elist\_2Elist\ A\_27b)})^{(((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2En}} \\
(((\forall V1f \in (((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum}).(\forall V2v0 \in \\
(ty\_2Elist\_2Elist\ A\_27c).(p\ (ap\ (ap\ (ap\ V0P\ V1f)\ (c\_2Elist\_2ENIL \\
A\_27b))\ V2v0)))){\wedge}((\forall V3f \in (((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum}). \\
(\forall V4v5 \in A\_27b.(\forall V5v6 \in (ty\_2Elist\_2Elist\ A\_27b). \\
(p\ (ap\ (ap\ (ap\ V0P\ V3f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V4v5)\ V5v6)) \\
(c\_2Elist\_2ENIL\ A\_27c)))))){\wedge}(\forall V6f \in (((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum}). \\
(\forall V7h1 \in A\_27b.(\forall V8t1 \in (ty\_2Elist\_2Elist\ A\_27b). \\
(\forall V9h2 \in A\_27c.(\forall V10t2 \in (ty\_2Elist\_2Elist\ A\_27c). \\
((p\ (ap\ (ap\ (ap\ V0P\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ ty\_2Enum\_2Enum\ ((A\_27a^{A\_27c})^{A\_27b}) \\
ty\_2Enum\_2Enum)\ V6f)\ \mathsf{c\_2Enum\_2ESUC}))\ V8t1)\ V10t2)){\Rightarrow}(p\ (ap\ (ap \\
(ap\ V0P\ V6f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V7h1)\ V8t1))\ (ap\ (ap \\
(c\_2Elist\_2ECONS\ A\_27c)\ V9h2)\ V10t2)))))))))))){\Rightarrow}(\forall V11v \in \\
(((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum}).(\forall V12v1 \in (ty\_2Elist\_2Elist \\
A\_27b).(\forall V13v2 \in (ty\_2Elist\_2Elist\ A\_27c).(p\ (ap\ (ap\ (ap \\
V0P\ V11v)\ V12v1)\ V13v2)))))))
\end{gathered} \tag{26}
$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow ((\forall V0v0 \in (ty\_2Elist\_2Elist\ A\_27c).(\forall V1f \in (((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum}).((ap\ (ap\ (ap\ (c\_2EindexedLists\_2EMAP2i\ A\_27a\ A\_27b\ A\_27c)\ V1f)\ (c\_2Elist\_2ENIL\ A\_27b))\ V0v0) = (c\_2Elist\_2ENIL\ A\_27a)))) \wedge ((\forall V2v6 \in (ty\_2Elist\_2Elist\ A\_27b).(\forall V3v5 \in A\_27b.(\forall V4f \in (((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum}).((ap\ (ap\ (ap\ (c\_2EindexedLists\_2EMAP2i\ A\_27a\ A\_27b\ A\_27c)\ V4f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V3v5)\ V2v6))\ (c\_2Elist\_2ENIL\ A\_27c)) = (c\_2Elist\_2ENIL\ A\_27a))))) \wedge (\forall V5t2 \in (ty\_2Elist\_2Elist\ A\_27c).(\forall V6t1 \in (ty\_2Elist\_2Elist\ A\_27b).(\forall V7h2 \in A\_27c.(\forall V8h1 \in A\_27b.(\forall V9f \in (((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum}). ((ap\ (ap\ (ap\ (c\_2EindexedLists\_2EMAP2i\ A\_27a\ A\_27b\ A\_27c)\ V9f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V8h1)\ V6t1))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27c)\ V7h2)\ V5t2)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ (ap\ (ap\ (ap\ V9f\ \mathsf{c\_2Enum\_2E0})\ V8h1)\ V7h2))\ (ap\ (ap\ (ap\ (c\_2EindexedLists\_2EMAP2i\ A\_27a\ A\_27b\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ ty\_2Enum\_2Enum\ ((A\_27a^{A\_27c})^{A\_27b})\ ty\_2Enum\_2Enum)\ V9f)\ \mathsf{c\_2Enum\_2ESUC}))\ V6t1)\ V5t2)))))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2EHD\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = V0h))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = \mathsf{c\_2Enum\_2E0}) \wedge (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t)))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1l \in A\_27b.(\forall V2ls \in (ty\_2Elist\_2Elist\ A\_27b).(((ap\ (c\_2Elist\_2EEL\ A\_27a)\ \mathsf{c\_2Enum\_2E0}) = (c\_2Elist\_2EHD\ A\_27a)) \wedge ((ap\ (ap\ (c\_2Elist\_2EEL\ A\_27b)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V1l)\ V2ls)) = (ap\ (ap\ (c\_2Elist\_2EEL\ A\_27b)\ V0n)\ V2ls)))))) \tag{30}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ \mathsf{c\_2Enum\_2E0})))) \tag{31}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)))) \tag{32}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in (((A\_27c^{A\_27b})^{A\_27a})^{ty\_2Enum\_2Enum}).\ (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27b).(\forall V3n \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V3n)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l1))) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V3n)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27b)\ V2l2)))) \Rightarrow ((ap\ (ap\ (c\_2Elist\_2EEL\ A\_27c)\ V3n)\ (ap\ (ap\ (ap\ (c\_2EindexedLists\_2EMAP2i\ A\_27c\ A\_27a\ A\_27b)\ V0f)\ V1l1)\ V2l2)) = (ap\ (ap\ (ap\ V0f\ V3n)\ (ap\ (ap\ (c\_2Elist\_2EEL\ A\_27a)\ V3n)\ V1l1))\ (ap\ (ap\ (c\_2Elist\_2EEL\ A\_27b)\ V3n)\ V2l2))))))))$$