

thm_2EindexedLists_2EFOLDER__MAPi
(TMR7Qqks6eqbcHg3WpKmas1dGmjFbJDEcwc)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (2)$$

Let $c_2EindexedLists_2EMAPi : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2EindexedLists_2EMAPi A_27a A_27b \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27b)})^{((A_27a^{A_27b})^{ty_2Enum_2Enum})}) \quad (3)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num ($

Definition 5 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{7}$$

Definition 6 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2EindexedLists_2EFOLDRi : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2EindexedLists_2EFOLDRi\ A_27a\ A_27b \in (((A_27a^{(ty_2Elist_2Elist\ A_27b)})_{A_27a})^{((A_27a^{A_27a})^{A_27b})^{ty_2Enum_2Enum}})) \tag{8}$$

Let $c_2Elist_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDR\ A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})_{A_27b})^{((A_27b^{A_27b})^{A_27a}})) \tag{9}$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})_{A_27a}) \tag{10}$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \tag{11}$$

Definition 7 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{13}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27a^{A.27c}). \\
& (\forall V2x \in A.27c.((ap\ (ap\ (ap\ (c.2Ecombin_2Eo\ A.27c\ A.27b\ A.27a) \\
& V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0f \in (A.27b^{A.27a}). \\
& (\forall V1g \in (A.27a^{A.27c}).(\forall V2h \in (A.27c^{A.27d}).((ap\ (\\
& ap\ (c.2Ecombin_2Eo\ A.27d\ A.27b\ A.27a)\ V0f)\ (ap\ (ap\ (c.2Ecombin_2Eo \\
& A.27d\ A.27a\ A.27c)\ V1g)\ V2h)) = (ap\ (ap\ (c.2Ecombin_2Eo\ A.27d\ A.27b \\
& A.27c)\ (ap\ (ap\ (c.2Ecombin_2Eo\ A.27c\ A.27b\ A.27a)\ V0f)\ V1g))\ V2h))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0f \in ((A.27a^{A.27b})^{ty_2Enum_2Enum}).((ap\ (ap\ (c.2EindexedLists_2EMAPi \\
& A.27a\ A.27b)\ V0f)\ (c.2Elist_2ENIL\ A.27b)) = (c.2Elist_2ENIL\ A.27a)))) \wedge \\
& (\forall V1f \in ((A.27a^{A.27b})^{ty_2Enum_2Enum}).(\forall V2h \in A.27b. \\
& (\forall V3t \in (ty_2Elist_2Elist\ A.27b).((ap\ (ap\ (c.2EindexedLists_2EMAPi \\
& A.27a\ A.27b)\ V1f)\ (ap\ (ap\ (c.2Elist_2ECONS\ A.27b)\ V2h)\ V3t)) = (ap \\
& (ap\ (c.2Elist_2ECONS\ A.27a)\ (ap\ (ap\ V1f\ c.2Enum_2E0)\ V2h))\ (ap\ (\\
& ap\ (c.2EindexedLists_2EMAPi\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Ecombin_2Eo \\
& ty_2Enum_2Enum\ (A.27a^{A.27b})\ ty_2Enum_2Enum)\ V1f)\ c.2Enum_2ESUC)) \\
& V3t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0f \in (((A.27a^{A.27a})^{A.27b})^{ty_2Enum_2Enum}).(\forall V1a \in \\
& A.27a.((ap\ (ap\ (ap\ (c.2EindexedLists_2EFOLDRi\ A.27a\ A.27b)\ V0f) \\
& V1a)\ (c.2Elist_2ENIL\ A.27b)) = V1a))) \wedge (\forall V2f \in (((A.27a^{A.27a})^{A.27b})^{ty_2Enum_2Enum}). \\
& (\forall V3a \in A.27a.(\forall V4h \in A.27b.(\forall V5t \in (ty_2Elist_2Elist \\
& A.27b).((ap\ (ap\ (ap\ (c.2EindexedLists_2EFOLDRi\ A.27a\ A.27b)\ V2f) \\
& V3a)\ (ap\ (ap\ (c.2Elist_2ECONS\ A.27b)\ V4h)\ V5t)) = (ap\ (ap\ (ap\ V2f\ c.2Enum_2E0) \\
& V4h)\ (ap\ (ap\ (ap\ (c.2EindexedLists_2EFOLDRi\ A.27a\ A.27b)\ (ap\ (ap \\
& (c.2Ecombin_2Eo\ ty_2Enum_2Enum\ ((A.27a^{A.27a})^{A.27b})\ ty_2Enum_2Enum) \\
& V2f)\ c.2Enum_2ESUC))\ V3a)\ V5t))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad (\forall V0f \in ((A.27b^{A.27b})^{A.27a}).(\forall V1e \in A.27b.((ap\ (\\
& \quad ap\ (ap\ (c.2Elist.2EFOLDR\ A.27a\ A.27b)\ V0f)\ V1e)\ (c.2Elist.2ENIL \\
& \quad A.27a)) = V1e))) \wedge (\forall V2f \in ((A.27b^{A.27b})^{A.27a}).(\forall V3e \in \\
& \quad A.27b.(\forall V4x \in A.27a.(\forall V5l \in (ty.2Elist.2Elist\ A.27a). \\
& \quad ((ap\ (ap\ (ap\ (c.2Elist.2EFOLDR\ A.27a\ A.27b)\ V2f)\ V3e)\ (ap\ (ap\ (c.2Elist.2ECONS \\
& \quad A.27a)\ V4x)\ V5l)) = (ap\ (ap\ V2f\ V4x)\ (ap\ (ap\ (ap\ (c.2Elist.2EFOLDR \\
& \quad A.27a\ A.27b)\ V2f)\ V3e)\ V5l))))))))) \\
& \hspace{15em} (20)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty.2Elist.2Elist\ A.27a)}). \\
& \quad (((p\ (ap\ V0P\ (c.2Elist.2ENIL\ A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\
& \quad A.27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& \quad c.2Elist.2ECONS\ A.27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\
& \quad A.27a).(p\ (ap\ V0P\ V3l)))))) \\
& \hspace{15em} (21)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow (\forall V0f \in ((A.27b^{A.27b})^{A.27a}).(\forall V1g \in \\
& \quad ((A.27a^{A.27c})^{ty.2Enum.2Enum}).(\forall V2a \in A.27b.(\forall V3l \in \\
& \quad (ty.2Elist.2Elist\ A.27c).((ap\ (ap\ (ap\ (c.2Elist.2EFOLDR\ A.27a \\
& \quad A.27b)\ V0f)\ V2a)\ (ap\ (ap\ (c.2EindexedLists.2EMAPi\ A.27a\ A.27c) \\
& \quad V1g)\ V3l)) = (ap\ (ap\ (ap\ (c.2EindexedLists.2EFOLDRi\ A.27b\ A.27c) \\
& \quad (ap\ (ap\ (c.2Ecombin.2Eo\ ty.2Enum.2Enum\ ((A.27b^{A.27b})^{A.27c})\ (\\
& \quad A.27a^{A.27c}))\ (ap\ (c.2Ecombin.2Eo\ A.27c\ (A.27b^{A.27b})\ A.27a)\ V0f)) \\
& \quad V1g))\ V2a)\ V3l)))))) \\
& \hspace{15em}
\end{aligned}$$