

thm_2EindexedLists_2EFOLDRI__APPEND
(TMb86bogxsXyQT6G7SVgNwvqz11pT4A76U3)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_7E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebool_2E_7F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7F))$

Definition 7 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{3}$$

Let $c_2EindexedLists_2EFOLDRI : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2EindexedLists_2EFOLDRI\ A_27a\ A_27b \in (((A_27a^{(ty_2Elist_2Elist\ A_27b)})^{A_27a})^{((A_27a^{A_27a})^{A_27b})^{ty_2Enum_2Enum}}) \tag{4}$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (5)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (10)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (11)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (12)$$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m)) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & ((ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Enum_2E0)\ V0m) = V0m) \wedge (((ap\ (\\ & ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m) \wedge (((ap\ (ap\ c_2Earithmetic_2E_2B \\ & (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & V0m)\ V1n))) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Enum_2ESUC \\ & V1n)) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n)))))) \end{aligned} \quad (14)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (15)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (16)$$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t) \Leftrightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap V0f V2x) = (ap V1g V2x)))))) \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (23)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (24)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27a^{A.27c}). \\
& (\forall V2x \in A.27c.((ap\ (ap\ (ap\ (c.2Ecombin_2Eo\ A.27c\ A.27b\ A.27a) \\
& V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0f \in (A.27b^{A.27a}). \\
& (\forall V1g \in (A.27a^{A.27c}).(\forall V2h \in (A.27c^{A.27d}).((ap\ (\\
& ap\ (c.2Ecombin_2Eo\ A.27d\ A.27b\ A.27a)\ V0f)\ (ap\ (ap\ (c.2Ecombin_2Eo \\
& A.27d\ A.27a\ A.27c)\ V1g)\ V2h)) = (ap\ (ap\ (c.2Ecombin_2Eo\ A.27d\ A.27b \\
& A.27c)\ (ap\ (ap\ (c.2Ecombin_2Eo\ A.27c\ A.27b\ A.27a)\ V0f)\ V1g))\ V2h))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0f \in (((A.27a^{A.27a})^{A.27b})^{ty_2Enum_2Enum}).(\forall V1a \in \\
& A.27a.((ap\ (ap\ (ap\ (c.2EindexedLists_2EFOLDRi\ A.27a\ A.27b)\ V0f) \\
& V1a)\ (c.2Elist_2ENIL\ A.27b)) = V1a))) \wedge (\forall V2f \in (((A.27a^{A.27a})^{A.27b})^{ty_2Enum_2Enum}). \\
& (\forall V3a \in A.27a.(\forall V4h \in A.27b.(\forall V5t \in (ty_2Elist_2Elist \\
& A.27b).((ap\ (ap\ (ap\ (c.2EindexedLists_2EFOLDRi\ A.27a\ A.27b)\ V2f) \\
& V3a)\ (ap\ (ap\ (c.2Elist_2ECONS\ A.27b)\ V4h)\ V5t)) = (ap\ (ap\ (ap\ V2f\ c.2Enum_2E0) \\
& V4h)\ (ap\ (ap\ (ap\ (c.2EindexedLists_2EFOLDRi\ A.27a\ A.27b)\ (ap\ (ap \\
& (c.2Ecombin_2Eo\ ty_2Enum_2Enum\ ((A.27a^{A.27a})^{A.27b})\ ty_2Enum_2Enum) \\
& V2f)\ c.2Enum_2ESUC))\ V3a)\ V5t))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\
& A.27a).((ap\ (ap\ (c.2Elist_2EAPPEND\ A.27a)\ (c.2Elist_2ENIL\ A.27a)) \\
& V0l) = V0l)) \wedge (\forall V1l1 \in (ty_2Elist_2Elist\ A.27a).(\forall V2l2 \in \\
& (ty_2Elist_2Elist\ A.27a).(\forall V3h \in A.27a.((ap\ (ap\ (c.2Elist_2EAPPEND \\
& A.27a)\ (ap\ (ap\ (c.2Elist_2ECONS\ A.27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap \\
& (c.2Elist_2ECONS\ A.27a)\ V3h)\ (ap\ (ap\ (c.2Elist_2EAPPEND\ A.27a) \\
& V1l1)\ V2l2))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (((ap\ (c.2Elist_2ELENGTH\ A.27a) \\
& (c.2Elist_2ENIL\ A.27a)) = c.2Enum_2E0) \wedge (\forall V0h \in A.27a.(\\
& \forall V1t \in (ty_2Elist_2Elist\ A.27a).((ap\ (c.2Elist_2ELENGTH \\
& A.27a)\ (ap\ (ap\ (c.2Elist_2ECONS\ A.27a)\ V0h)\ V1t)) = (ap\ c.2Enum_2ESUC \\
& (ap\ (c.2Elist_2ELENGTH\ A.27a)\ V1t))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}), \\
& (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& A_27a).(p\ (ap\ V0P\ V1t))) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ A_27a\ V2h\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A_27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{30}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0a \in A_27b. (\forall V1l1 \in (ty_2Elist_2Elist\ A_27a). (\\
& \forall V2l2 \in (ty_2Elist_2Elist\ A_27a). (\forall V3f \in (((A_27b^{A_27b})^{A_27a})^{ty_2Enum_2Enum}). \\
& ((ap\ (ap\ (ap\ (c_2EindexedLists_2EFOLDRi\ A_27b\ A_27a)\ V3f)\ V0a) \\
& (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V1l1)\ V2l2)) = (ap\ (ap\ (ap\ (c_2EindexedLists_2EFOLDRi \\
& A_27b\ A_27a)\ V3f)\ (ap\ (ap\ (ap\ (c_2EindexedLists_2EFOLDRi\ A_27b \\
& A_27a)\ (ap\ (ap\ (c_2Ecombin_2Eo\ ty_2Enum_2Enum\ ((A_27b^{A_27b})^{A_27a}) \\
& ty_2Enum_2Enum)\ V3f)\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (c_2Elist_2ELENGTH \\
& A_27a)\ V1l1))))))\ V0a)\ V2l2))\ V1l1))))))
\end{aligned}$$