

# thm\_2EindexedLists\_2EFOLDRI\_\_CONG (TMS5TKbX3DA3vC2nSKZ2VnmTwKy3gc9sj4R)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p (ap P x))$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a))))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))))$

**Definition 6** We define  $c\_2Ecombin\_2E\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g \in (A\_27c^{A\_27b}).$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2EindexedLists\_2EFOLDRI : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2EindexedLists\_2EFOLDRI A\_27a A\_27b \in (((A\_27a^{(ty\_2Elist\_2Elist A\_27b)})_{A\_27a})(((A\_27a^{A\_27b})^{A\_27b})^{ty\_2Enum\_2Enum})) \quad (3)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ELENGTH A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist A\_27a)}) \quad (4)$$

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (5)$$

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (6)$$

Let  $c\_2Elist\_2ELIST\_TO\_SET : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_TO\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (7)$$

**Definition 10** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

**Definition 12** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (10)$$

**Definition 13** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ V0m)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (11)$$

**Definition 14** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 15** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m\ V1n)$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (16) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (17) \end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg ( \\ & p V0t)))))) \quad (20) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in \\ & (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A\_27a. (p (ap V0P V3x))) \wedge (\forall V4x \in A\_27a. (p ( \\ & ap V1Q V4x)))))) \quad (21) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in \\ & 2. ((\forall V2x \in A\_27a. ((p (ap V0P V2x)) \Rightarrow (p V1Q))) \Leftrightarrow ((\exists V3x \in \\ & A\_27a. (p (ap V0P V3x)) \Rightarrow (p V1Q)))))) \quad (22) \end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (25)$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2R \in 2.(((p V0P) \vee (p V1Q)) \Rightarrow (p V2R)) \Leftrightarrow (((p V0P) \Rightarrow (p V2R)) \wedge ((p V1Q) \Rightarrow (p V2R)))))) \quad (26)$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2R \in 2.(((p V0P) \Rightarrow ((p V1Q) \wedge (p V2R))) \Leftrightarrow (((p V0P) \Rightarrow (p V1Q)) \wedge ((p V0P) \Rightarrow (p V2R)))))) \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (28)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (29)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}).(\forall V1v \in A_{.27a}.((\forall V2x \in A_{.27a}.((V2x = V1v) \Rightarrow (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (30)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}.nonempty A_{.27c} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1g \in (A_{.27a}^{A_{.27c}}).(\forall V2x \in A_{.27c}.((ap (ap (ap (c.2Ecombin_2Eo A_{.27c} A_{.27b} A_{.27a}) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \quad (31)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) (ap c\_2Enum\_2ESUC V1m))) \Leftrightarrow ( \\
& (V0n = c\_2Enum\_2E0) \vee (\exists V2n0 \in ty\_2Enum\_2Enum. ((V0n = (ap \\
& c\_2Enum\_2ESUC V2n0)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C V2n0) V1m))))))))) \\
& \tag{32}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\
& (\forall V0f \in (((A\_27a^{A\_27a})^{A\_27b})^{ty\_2Enum\_2Enum}). (\forall V1a \in \\
& A\_27a. ((ap (ap (ap (c\_2EindexedLists\_2EFOLDRi A\_27a A\_27b) V0f) \\
V1a) (c\_2Elist\_2ENIL A\_27b)) = V1a))) \wedge (\forall V2f \in (((A\_27a^{A\_27a})^{A\_27b})^{ty\_2Enum\_2Enum}). \\
& (\forall V3a \in A\_27a. (\forall V4h \in A\_27b. (\forall V5t \in (ty\_2Elist\_2Elist \\
& A\_27b). ((ap (ap (ap (c\_2EindexedLists\_2EFOLDRi A\_27a A\_27b) V2f) \\
V3a) (ap (ap (c\_2Elist\_2ECONS A\_27b) V4h) V5t)) = (ap (ap (ap V2f c\_2Enum\_2E0) \\
& V4h) (ap (ap (ap (c\_2EindexedLists\_2EFOLDRi A\_27a A\_27b) (ap (ap \\
& (c\_2Ecombin\_2Eo ty\_2Enum\_2Enum ((A\_27a^{A\_27a})^{A\_27b}) ty\_2Enum\_2Enum) \\
& V2f) c\_2Enum\_2ESUC)) V3a) V5t))))))))) \\
& \tag{33}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (((ap (c\_2Elist\_2ELENGTH A\_27a) \\
& (c\_2Elist\_2ENIL A\_27a)) = c\_2Enum\_2E0) \wedge (\forall V0h \in A\_27a. ( \\
& \forall V1t \in (ty\_2Elist\_2Elist A\_27a). ((ap (c\_2Elist\_2ELENGTH \\
A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V0h) V1t)) = (ap c\_2Enum\_2ESUC \\
& (ap (c\_2Elist\_2ELENGTH A\_27a) V1t)))))) \\
& \tag{34}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist A\_27a)}). \\
& (((p (ap V0P (c\_2Elist\_2ENIL A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\
& A\_27a). ((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A\_27a. (p (ap V0P (ap (ap ( \\
& c\_2Elist\_2ECONS A\_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\
& A\_27a). (p (ap V0P V3l)))))) \\
& \tag{35}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow ((\forall V0x \in A\_27a. ((p (ap (ap \\
& (c\_2Ebool\_2EIN A\_27a) V0x) (ap (c\_2Elist\_2ELIST\_TO\_SET A\_27a) \\
& (c\_2Elist\_2ENIL A\_27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A\_27a. (\forall V2h \in \\
& A\_27a. (\forall V3t \in (ty\_2Elist\_2Elist A\_27a). ((p (ap (ap (c\_2Ebool\_2EIN \\
A\_27a) V1x) (ap (c\_2Elist\_2ELIST\_TO\_SET A\_27a) (ap (ap (c\_2Elist\_2ECONS \\
& A\_27a) V2h) V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p (ap (ap (c\_2Ebool\_2EIN A\_27a) \\
& V1x) (ap (c\_2Elist\_2ELIST\_TO\_SET A\_27a) V3t))))))))) \\
& \tag{36}
\end{aligned}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) c\_2Enum\_2E0)))) \quad (37)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Enum\_2ESUC V0n)))) \quad (38)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V1l2 \in (ty\_2Elist\_2Elist \\ & \quad A\_27a). (\forall V2f1 \in (((A\_27b^{A\_27b})^{A\_27a})^{ty\_2Enum\_2Enum}). \\ & \quad (\forall V3f2 \in (((A\_27b^{A\_27b})^{A\_27a})^{ty\_2Enum\_2Enum}). (\forall V4a1 \in \\ & \quad A\_27b. (\forall V5a2 \in A\_27b. ((V0l1 = V1l2) \Rightarrow ((\forall V6n \in ty\_2Enum\_2Enum. \\ & \quad (\forall V7e \in A\_27a. (\forall V8a \in A\_27b. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\ & V6n) (ap (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l2))) \Rightarrow ((p (ap (ap (c\_2Ebool\_2EIN \\ & \quad A\_27a)\ V7e) (ap (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ V1l2))) \Rightarrow ((ap \\ & \quad (ap (ap V2f1\ V6n)\ V7e)\ V8a) = (ap (ap (ap V3f2\ V6n)\ V7e)\ V8a)))))) \Rightarrow \\ & ((V4a1 = V5a2) \Rightarrow ((ap (ap (ap (c\_2EindexedLists\_2EFOLDRi\ A\_27b\ A\_27a) \\ & \quad V2f1)\ V4a1)\ V0l1) = (ap (ap (ap (c\_2EindexedLists\_2EFOLDRi\ A\_27b \\ & \quad A\_27a)\ V3f2)\ V5a2)\ V1l2)))))))))) \end{aligned}$$