thm_2EindexedLists_2ELIST__RELi__LENGTH (TMQES-CUZHH4ZbS8BVtZWfCvSbvAifuHnLzs)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (2)$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (3)$$

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (4)$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (5)$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (6)$$

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$*.*

Definition 7 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 A

Definition 8 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in 2$

Definition 9 *We define* c_2EindexedLists_2ELIST__RELi *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0R \in (((2^{A\_27b})^{A\_27}$

Definition 10 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (7)$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (8)$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (9)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (10)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (11)$$

Definition 12 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 13 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 14 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 15 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 16 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (12)$$

Definition 17 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 18 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 19 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Definition 20 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Definition 21 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 22 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{15}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{16}$$

Definition 23 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 24 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) \Leftrightarrow (V0m = V1n))))) \tag{17}$$

Assume the following.

$$True \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{23}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{24}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in (((2^{A\_27b})^{A\_27a})^{ty\_2Enum\_2Enum}).(\forall V1LIST\_\_RELi\_27 \in ((2^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ (ap\ V1LIST\_\_RELi\_27\ (c\_2Elist\_2ENIL\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27b))) \wedge (\forall V2h1 \in A\_27a.(\forall V3h2 \in A\_27b.(\forall V4l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V5l2 \in (ty\_2Elist\_2Elist\ A\_27b).(((p\ (ap\ (ap\ (ap\ V0R\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V4l1))\ V2h1)\ V3h2)) \wedge ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EindexedLists\_2ELIST\_\_RELi}\ A\_27a\ A\_27b)\ V0R)\ V4l1)\ V5l2)) \wedge (p\ (ap\ (ap\ V1LIST\_\_RELi\_27\ V4l1)\ V5l2)))) \Rightarrow (p\ (ap\ (ap\ V1LIST\_\_RELi\_27\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V4l1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h1)\ (c\_2Elist\_2ENIL\ A\_27a))))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27b)\ V5l2)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V3h2)\ (c\_2Elist\_2ENIL\ A\_27b)))))))))) \Rightarrow (\forall V6a0 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V7a1 \in (ty\_2Elist\_2Elist\ A\_27b).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EindexedLists\_2ELIST\_\_RELi}\ A\_27a\ A\_27b)\ V0R)\ V6a0)\ V7a1)) \Rightarrow (p\ (ap\ (ap\ V1LIST\_\_RELi\_27\ V6a0)\ V7a1)))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = \mathsf{c\_2Enum\_2E0}) \wedge (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t)))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V1l2)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1))\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l2))))) \tag{28}$$

Assume the following.

$$\begin{gathered}
((\forall V0n \in \mathit{ty_2Enum_2Enum}.((ap\ (ap\ \mathit{c_2Earithmetic_2E_2B} \\
\mathsf{c_2Enum_2E0})\ V0n) = V0n))\wedge((\forall V1n \in \mathit{ty_2Enum_2Enum}.((ap \\
(ap\ \mathit{c_2Earithmetic_2E_2B}\ V1n)\ \mathsf{c_2Enum_2E0}) = V1n))\wedge((\forall V2n \in \\
\mathit{ty_2Enum_2Enum}.(\forall V3m \in \mathit{ty_2Enum_2Enum}.((ap\ (ap\ \mathit{c_2Earithmetic_2E_2B} \\
(ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V2n))\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
V3m)) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Enumeral_2EiZ}\ (ap \\
(ap\ \mathit{c_2Earithmetic_2E_2B}\ V2n)\ V3m))))))\wedge((\forall V4n \in \mathit{ty_2Enum_2Enum}. \\
((ap\ (ap\ \mathit{c_2Earithmetic_2E_2A}\ \mathsf{c_2Enum_2E0})\ V4n) = \mathsf{c_2Enum_2E0}))\wedge \\
((\forall V5n \in \mathit{ty_2Enum_2Enum}.((ap\ (ap\ \mathit{c_2Earithmetic_2E_2A} \\
V5n)\ \mathsf{c_2Enum_2E0}) = \mathsf{c_2Enum_2E0}))\wedge((\forall V6n \in \mathit{ty_2Enum_2Enum}. \\
(\forall V7m \in \mathit{ty_2Enum_2Enum}.((ap\ (ap\ \mathit{c_2Earithmetic_2E_2A}\ (\\
ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V6n))\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
V7m)) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ (ap\ \mathit{c_2Earithmetic_2E_2A} \\
V6n)\ V7m)))))\wedge((\forall V8n \in \mathit{ty_2Enum_2Enum}.((ap\ (ap\ \mathit{c_2Earithmetic_2E_2D} \\
\mathsf{c_2Enum_2E0})\ V8n) = \mathsf{c_2Enum_2E0}))\wedge((\forall V9n \in \mathit{ty_2Enum_2Enum}. \\
((ap\ (ap\ \mathit{c_2Earithmetic_2E_2D}\ V9n)\ \mathsf{c_2Enum_2E0}) = V9n))\wedge((\forall V10n \in \\
\mathit{ty_2Enum_2Enum}.(\forall V11m \in \mathit{ty_2Enum_2Enum}.((ap\ (ap\ \mathit{c_2Earithmetic_2E_2D} \\
(ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V10n))\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
V11m)) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ (ap\ \mathit{c_2Earithmetic_2E_2D} \\
V10n)\ V11m)))))\wedge((\forall V12n \in \mathit{ty_2Enum_2Enum}.((ap\ (ap\ \mathit{c_2Earithmetic_2EEXP} \\
\mathsf{c_2Enum_2E0})\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
V12n))) = \mathsf{c_2Enum_2E0}))\wedge((\forall V13n \in \mathit{ty_2Enum_2Enum}.((ap \\
(ap\ \mathit{c_2Earithmetic_2EEXP}\ \mathsf{c_2Enum_2E0})\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V13n))) = \mathsf{c_2Enum_2E0}))\wedge((\forall V14n \in \\
\mathit{ty_2Enum_2Enum}.((ap\ (ap\ \mathit{c_2Earithmetic_2EEXP}\ V14n)\ \mathsf{c_2Enum_2E0}) = \\
(ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO}))))\wedge \\
((\forall V15n \in \mathit{ty_2Enum_2Enum}.(\forall V16m \in \mathit{ty_2Enum_2Enum}. \\
((ap\ (ap\ \mathit{c_2Earithmetic_2EEXP}\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V15n)) \\
(ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V16m)) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ (ap\ \mathit{c_2Earithmetic_2EEXP}\ V15n)\ V16m)))))\wedge(((ap\ \mathsf{c_2Enum_2ESUC} \\
\mathsf{c_2Enum_2E0}) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
\mathsf{c_2Earithmetic_2EZERO})))\wedge((\forall V17n \in \mathit{ty_2Enum_2Enum}.(\\
(ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V17n)) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ \mathsf{c_2Enum_2ESUC}\ V17n))))\wedge(((ap\ \mathsf{c_2Eprim__rec_2EPRE}\ \mathsf{c_2Enum_2E0}) = \\
\mathsf{c_2Enum_2E0})\wedge((\forall V18n \in \mathit{ty_2Enum_2Enum}.((ap\ \mathsf{c_2Eprim__rec_2EPRE} \\
(ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V18n)) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ \mathsf{c_2Eprim__rec_2EPRE}\ V18n))))\wedge((\forall V19n \in \mathit{ty_2Enum_2Enum}. \\
(((ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V19n) = \mathsf{c_2Enum_2E0})\Leftrightarrow(V19n = \mathsf{c_2Earithmetic_2EZERO})))\wedge \\
((\forall V20n \in \mathit{ty_2Enum_2Enum}.((\mathsf{c_2Enum_2E0} = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
V20n))\Leftrightarrow(V20n = \mathsf{c_2Earithmetic_2EZERO})))\wedge((\forall V21n \in \mathit{ty_2Enum_2Enum}. \\
(\forall V22m \in \mathit{ty_2Enum_2Enum}.(((ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
V21n) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge \\
((\forall V23n \in \mathit{ty_2Enum_2Enum}.((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C} \\
V23n)\ \mathsf{c_2Enum_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in \mathit{ty_2Enum_2Enum}. \\
((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ \mathsf{c_2Enum_2E0})\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
V24n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ \mathsf{c_2Earithmetic_2EZERO}) \\
V24n))))\wedge((\forall V25n \in \mathit{ty_2Enum_2Enum}.(\forall V26m \in \mathit{ty_2Enum_2Enum}. \\
((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
V25n))\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C} \\
V25n)\ V26m)))))\wedge((\forall V27n \in \mathit{ty_2Enum_2Enum}.((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3E} \\
\mathsf{c_2Enum_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in \mathit{ty_2Enum_2Enum}. \\
((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3E}\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
V28n))\ \mathsf{c_2Enum_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ \mathsf{c_2Earithmetic_2EZERO}) \\
V28n))))\wedge((\forall V29n \in \mathit{ty_2Enum_2Enum}.(\forall V30m \in \mathit{ty_2Enum_2Enum}. \\
((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3E}\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
V29n))\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C} \\
V30m)\ V29n)))))\wedge((\forall V31n \in \mathit{ty_2Enum_2Enum}.((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D} \\
\mathsf{c_2Enum_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in \mathit{ty_2Enum_2Enum}. \\
((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}
\end{gathered}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in (((2^{A\_27b})^{A\_27a})^{ty\_2Enum\_2Enum}).(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27b). ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EindexedLists\_2ELIST\_\_RELi}\ A\_27a\ A\_27b)\ V0R)\ V1l1)\ V2l2)) \Rightarrow ((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l1) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27b)\ V2l2))))))$$