

# thm\_2EindexedLists\_2ELIST\_\_RELi\_\_rules (TMJn1kT4hqjX1c8CXwNShgerfYkBGVayzjm)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{2}$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{3}$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{4}$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{5}$$



**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0R \in (((2^{A\_27b})^{A\_27a})^{ty\_2Enum\_2Enum}).((p\ (ap\ (ap\ (ap \\ & (c\_2IndexedLists\_2ELIST\_RELi\ A\_27a\ A\_27b)\ V0R)\ (c\_2Elist\_2ENIL \\ & A\_27a))\ (c\_2Elist\_2ENIL\ A\_27b))) \wedge (\forall V1h1 \in A\_27a. (\forall V2h2 \in \\ & A\_27b. (\forall V3l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V4l2 \in \\ & (ty\_2Elist\_2Elist\ A\_27b). (((p\ (ap\ (ap\ (ap\ V0R\ (ap\ (c\_2Elist\_2ELENGTH \\ & A\_27a)\ V3l1))\ V1h1)\ V2h2)) \wedge (p\ (ap\ (ap\ (ap\ (c\_2IndexedLists\_2ELIST\_RELi \\ & A\_27a\ A\_27b)\ V0R)\ V3l1)\ V4l2))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2IndexedLists\_2ELIST\_RELi \\ & A\_27a\ A\_27b)\ V0R)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V3l1)\ (ap\ (ap \\ & (c\_2Elist\_2ECONS\ A\_27a)\ V1h1)\ (c\_2Elist\_2ENIL\ A\_27a))))\ (ap\ ( \\ & ap\ (c\_2Elist\_2EAPPEND\ A\_27b)\ V4l2)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b) \\ & V2h2)\ (c\_2Elist\_2ENIL\ A\_27b))))))))))))) \end{aligned}$$