

thm\_2IndexedLists\_2EMAP2i\_\_NIL2  
(TMXYKyAGWc6GvBwg57Lm8FMTu6QMN9XmtrM)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o(x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))) (\lambda V1Q \in 2.V1Q)))$ .

**Definition 4** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (c\_2ESUC\_REP m))$ .

**Definition 5** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.\lambda V0f \in (A-27b^{A-27c}).\lambda V1g \in (A-27c^{A-27b}).(ap (ap (c\_2Ebool\_2E\_21 (2^{A-27a}))) (\lambda V2h \in 2.V2h)) (\lambda V3i \in 2.V3i)$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{5}$$

**Definition 6** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (6)$$

Let  $c\_2EindexedLists\_2EMAP2i : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow c\_2EindexedLists\_2EMAP2i\ A\_27a\ A\_27b\ A\_27c \in ( \\ & (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27c)}(ty\_2Elist\_2Elist\ A\_27b))^{((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum}})) \end{aligned} \quad (7)$$

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (8)$$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A \wedge p\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a)\ P)))$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (9)$$

**Definition 11** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow ((\forall V0v0 \in (ty\_2Elist\_2Elist\ A.27c).(\forall V1f \in \\
& (((A.27a^{A.27c})^{A.27b})^{ty\_2Enum\_2Enum}).((ap\ (ap\ (ap\ (c.2EindexedLists\_2EMAP2i \\
& A.27a\ A.27b\ A.27c)\ V1f)\ (c.2Elist\_2ENIL\ A.27b))\ V0v0) = (c.2Elist\_2ENIL \\
& A.27a)))) \wedge ((\forall V2v6 \in (ty\_2Elist\_2Elist\ A.27b).(\forall V3v5 \in \\
& A.27b.(\forall V4f \in (((A.27a^{A.27c})^{A.27b})^{ty\_2Enum\_2Enum}).( \\
& (ap\ (ap\ (ap\ (c.2EindexedLists\_2EMAP2i\ A.27a\ A.27b\ A.27c)\ V4f)\ ( \\
& ap\ (ap\ (c.2Elist\_2ECONS\ A.27b)\ V3v5)\ V2v6))\ (c.2Elist\_2ENIL\ A.27c)) = \\
& (c.2Elist\_2ENIL\ A.27a)))) \wedge (\forall V5t2 \in (ty\_2Elist\_2Elist \\
& A.27c).(\forall V6t1 \in (ty\_2Elist\_2Elist\ A.27b).(\forall V7h2 \in \\
& A.27c.(\forall V8h1 \in A.27b.(\forall V9f \in (((A.27a^{A.27c})^{A.27b})^{ty\_2Enum\_2Enum}). \\
& ((ap\ (ap\ (ap\ (c.2EindexedLists\_2EMAP2i\ A.27a\ A.27b\ A.27c)\ V9f) \\
& (ap\ (ap\ (c.2Elist\_2ECONS\ A.27b)\ V8h1)\ V6t1))\ (ap\ (ap\ (c.2Elist\_2ECONS \\
& A.27c)\ V7h2)\ V5t2)) = (ap\ (ap\ (c.2Elist\_2ECONS\ A.27a)\ (ap\ (ap\ (ap \\
& V9f\ c.2Enum\_2E0)\ V8h1)\ V7h2))\ (ap\ (ap\ (ap\ (c.2EindexedLists\_2EMAP2i \\
& A.27a\ A.27b\ A.27c)\ (ap\ (ap\ (c.2Ecombin\_2Eo\ ty\_2Enum\_2Enum\ ((A.27a^{A.27c})^{A.27b}) \\
& ty\_2Enum\_2Enum)\ V9f)\ c.2Enum\_2ESUC))\ V6t1)\ V5t2)))))))))) \\
& \hspace{15em} (12)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\
& A.27a).((V0l = (c.2Elist\_2ENIL\ A.27a)) \vee (\exists V1h \in A.27a.( \\
& \exists V2t \in (ty\_2Elist\_2Elist\ A.27a).(V0l = (ap\ (ap\ (c.2Elist\_2ECONS \\
& A.27a)\ V1h)\ V2t)))))) \hspace{10em} (13)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in (((A.27a^{A.27c})^{A.27b})^{ty\_2Enum\_2Enum}). \\
& (\forall V1l1 \in (ty\_2Elist\_2Elist\ A.27b).((ap\ (ap\ (ap\ (c.2EindexedLists\_2EMAP2i \\
& A.27a\ A.27b\ A.27c)\ V0f)\ V1l1)\ (c.2Elist\_2ENIL\ A.27c)) = (c.2Elist\_2ENIL \\
& A.27a))))))
\end{aligned}$$