thm_2EindexedLists_2EMAP2ia__NIL2 (TMUxK-tYeb5xe2fvymBBGWk5NKyAMcUiw2qL)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 3 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 4 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{7}$$

Let $c\_2EindexedLists\_2EMAP2ia : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow c\_2EindexedLists\_2EMAP2ia\ A\_27a\ A\_27b\ A\_27c \in (((((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27c)})^{(ty\_2Elist\_2Elist\ A\_27b)})^{ty\_2Enum\_2Enum})^{(((A\_27a^{A\_27c})^{A\_27b} \tag{8}$$

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{9}$$

Definition 11 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 12 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{10}$$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow ((\forall V0v0 \in (ty\_2Elist\_2Elist\ A\_27c).(\forall V1i \in ty\_2Enum\_2Enum.(\forall V2f \in (((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum}).\ ((ap\ (ap\ (ap\ (ap\ (c\_2EindexedLists\_2EMAP2ia\ A\_27a\ A\_27b\ A\_27c)\ V2f)\ V1i)\ (c\_2Elist\_2ENIL\ A\_27b))\ V0v0) = (c\_2Elist\_2ENIL\ A\_27a))))) \wedge ((\forall V3v8 \in (ty\_2Elist\_2Elist\ A\_27b).(\forall V4v7 \in A\_27b.\ (\forall V5i \in ty\_2Enum\_2Enum.(\forall V6f \in (((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum}).\ ((ap\ (ap\ (ap\ (ap\ (c\_2EindexedLists\_2EMAP2ia\ A\_27a\ A\_27b\ A\_27c)\ V6f)\ V5i)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V4v7)\ V3v8))\ (c\_2Elist\_2ENIL\ A\_27c)) = (c\_2Elist\_2ENIL\ A\_27a)))))) \wedge (\forall V7t2 \in (ty\_2Elist\_2Elist\ A\_27c).(\forall V8t1 \in (ty\_2Elist\_2Elist\ A\_27b).(\forall V9i \in ty\_2Enum\_2Enum.(\forall V10h2 \in A\_27c.(\forall V11h1 \in A\_27b.\ (\forall V12f \in (((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum}).((ap\ (ap\ (ap\ (ap\ (c\_2EindexedLists\_2EMAP2ia\ A\_27a\ A\_27b\ A\_27c)\ V12f)\ V9i)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V11h1)\ V8t1))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27c)\ V10h2)\ V7t2)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ (ap\ (ap\ (ap\ V12f\ V9i)\ V11h1)\ V10h2))\ (ap\ (ap\ (ap\ (ap\ (c\_2EindexedLists\_2EMAP2ia\ A\_27a\ A\_27b\ A\_27c)\ V12f)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V9i)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V8t1)\ V7t2))))))))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((V0l = (c\_2Elist\_2ENIL\ A\_27a)) \vee (\exists V1h \in A\_27a.(\ \exists V2t \in (ty\_2Elist\_2Elist\ A\_27a).(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1h)\ V2t)))))) \tag{14}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in (((A\_27a^{A\_27c})^{A\_27b})^{ty\_2Enum\_2Enum}).\ (\forall V1i \in ty\_2Enum\_2Enum.(\forall V2l1 \in (ty\_2Elist\_2Elist\ A\_27b).((ap\ (ap\ (ap\ (ap\ (c\_2EindexedLists\_2EMAP2ia\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ V1i)\ V2l1)\ (c\_2Elist\_2ENIL\ A\_27c)) = (c\_2Elist\_2ENIL\ A\_27a)))))$$