

thm_2EindexedLists_2EMAPi_APPEND
(TMHYw7yofonqTL5bQZ1Mqx1r3H8MgQBrbfx)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_{\text{2Ebool_2E_21}}$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A \rightarrow 27a}).(ap\ (ap\ (ap\ (c_{\text{2Emin_2E_3D}}\ (2^{A \rightarrow 27a})\ P)\ V)\ 0)\ P)$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21 2))(\lambda V2t \in 2.$

Definition 6 We define $c \in \text{Ecombin_2Eo}$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in (A.27b^{A.27c}).\lambda V1g$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \text{nonempty } A \Rightarrow \text{nonempty} (\text{ty_2Elist_2Elist } A) \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

nonempty *ty_2Enum_2Enum* (2)

Let $c_2EindexedLists_2EM API : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2E\text{indexedLists}_2EM API \\ A_27a \ A_27b \in (((ty_2Elist_2Elist \ A_27a)^{(ty_2Elist_2Elist \ A_27b)})^{((A_27a^A)^{A_27b})^{ty_2Eenum_2Eenum}}) \quad (3)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (\text{ty_2Enum_2Enum}(\text{ty_2Elist_2Elist } A_27a)) \\ (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A \exists a. \text{nonempty } A \Rightarrow c \in \text{Elist_ENIL } A \in (\text{ty_Elist_Elist} \\ A) \quad (5)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (6)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (7)$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (9)$$

Definition 7 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$.

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (12)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$.

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p(x))) \text{ of type } \iota \Rightarrow \iota$.

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ P)))$.

Definition 13 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ (c_2Eprim_rec_2E_3C\ m)\ n)$.

Definition 14 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ (c_2Earithmetic_2E_3E\ m)\ n)$.

Definition 15 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ (c_2Earithmetic_2E_3E_3D\ m)\ n)$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 18 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2EEEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (15)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (16)$$

Definition 19 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 20 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Enum_2ESUC\ (ap\$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (17)$$

Definition 21 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 22 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E$

Definition 23 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E$

Definition 24 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 25 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.($

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m)) \quad (18)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V0m)))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\ & (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) V2p))))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\ & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\ & (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\ & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\ & V0m) V1n)))))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\ & V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\ & V1n)) V0m))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\ & c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO))) V0n))) \end{aligned} \quad (25)$$

Assume the following.

$$True \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0t \in 2.((\forall V1x \in \\ A_{27a}.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))) \quad (30)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((V0x = V0x) \Leftrightarrow True)) \quad (31)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1y \in \\ A_{27a}.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & \forall A_{27b}.nonempty\ A_{27b} \Rightarrow \\ & \forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1g \in (A_{27b}^{A_{27a}}).((V0f = \\ V1g) \Leftrightarrow (\forall V2x \in A_{27a}.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a &\Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\ \text{nonempty } A_27c &\Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}). \\ (\forall V2x \in A_27c.((ap (ap (ap (c_2Ecombin_2Eo A_27c A_27b A_27a) \\ V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a &\Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\ \text{nonempty } A_27c &\Rightarrow (\forall A_27d.\text{nonempty } A_27d \Rightarrow (\forall V0f \in (A_27b^{A_27a}). \\ (\forall V1g \in (A_27a^{A_27c}).(\forall V2h \in (A_27c^{A_27d}).((ap (\\ ap (c_2Ecombin_2Eo A_27d A_27b A_27a) V0f) (ap (ap (c_2Ecombin_2Eo \\ A_27d A_27a A_27c) V1g) V2h)) = (ap (ap (c_2Ecombin_2Eo A_27d A_27b \\ A_27c) (ap (ap (c_2Ecombin_2Eo A_27c A_27b A_27a) V0f) V1g)) V2h))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a &\Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\ (\forall V0f \in ((A_27a^{A_27b})^{ty_2Enum_2Enum}).((ap (ap (c_2EindexedLists_2EMAPi \\ A_27a A_27b) V0f) (c_2Elist_2ENIL A_27b)) = (c_2Elist_2ENIL A_27a))) \wedge \\ (\forall V1f \in ((A_27a^{A_27b})^{ty_2Enum_2Enum}).(\forall V2h \in A_27b. \\ (\forall V3t \in (ty_2Elist_2Elist A_27b).((ap (ap (c_2EindexedLists_2EMAPi \\ A_27a A_27b) V1f) (ap (ap (c_2Elist_2ECONS A_27b) V2h) V3t)) = (ap (\\ ap (c_2Elist_2ECONS A_27a) (ap (ap V1f c_2Enum_2E0) V2h)) (ap (\\ ap (c_2EindexedLists_2EMAPi A_27a A_27b) (ap (ap (c_2Ecombin_2Eo \\ ty_2Enum_2Enum (A_27a^{A_27b}) ty_2Enum_2Enum) V1f) c_2Enum_2ESUC) \\ V3t))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a &\Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\ A_27a).((ap (ap (c_2Elist_2EAPPEND A_27a) (c_2Elist_2ENIL A_27a)) \\ V0l) = V0l)) \wedge (\forall V1l1 \in (ty_2Elist_2Elist A_27a).(\forall V2l2 \in \\ (ty_2Elist_2Elist A_27a).(\forall V3h \in A_27a.((ap (ap (c_2Elist_2EAPPEND \\ A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V3h) V1l1)) V2l2) = (ap (ap \\ (c_2Elist_2ECONS A_27a) V3h) (ap (ap (c_2Elist_2EAPPEND A_27a) \\ V1l1) V2l2))))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a &\Rightarrow (((ap (c_2Elist_2ELENGTH A_27a) \\ (c_2Elist_2ENIL A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a.(\forall V1t \in (ty_2Elist_2Elist A_27a).((ap (c_2Elist_2ELENGTH \\ A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V0h) V1t)) = (ap c_2Enum_2ESUC \\ (ap (c_2Elist_2ELENGTH A_27a) V1t))))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned}
 & \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist A_{\text{27a}})}). \\
 & (((p (ap V0P (c_2Elist_2ENIL A_{\text{27a}}))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
 & A_{\text{27a}}).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A_{\text{27a}}.(p (ap V0P (ap (ap \\
 & c_2Elist_2ECONS A_{\text{27a}}) V2h) V1t))))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
 & A_{\text{27a}}).(p (ap V0P V3l)))))) \\
 & (41)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0a0 \in A_{\text{27a}}.(\forall V1a1 \in \\
 & (ty_2Elist_2Elist A_{\text{27a}}).(\forall V2a0_27 \in A_{\text{27a}}.(\forall V3a1_27 \in \\
 & (ty_2Elist_2Elist A_{\text{27a}}).(((ap (ap (c_2Elist_2ECONS A_{\text{27a}}) V0a0) \\
 & V1a1) = (ap (ap (c_2Elist_2ECONS A_{\text{27a}}) V2a0_27) V3a1_27)) \Leftrightarrow ((V0a0 = \\
 & V2a0_27) \wedge (V1a1 = V3a1_27))))))) \\
 & (42)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow ((\forall V0l1 \in (ty_2Elist_2Elist \\
 & A_{\text{27a}}).(\forall V1l2 \in (ty_2Elist_2Elist A_{\text{27a}}).(\forall V2l3 \in \\
 & (ty_2Elist_2Elist A_{\text{27a}}).(((ap (ap (c_2Elist_2EAPPEND A_{\text{27a}}) \\
 & V0l1) V1l2) = (ap (ap (c_2Elist_2EAPPEND A_{\text{27a}}) V0l1) V2l3)) \Leftrightarrow (V1l2 = \\
 & V2l3))) \wedge (\forall V3l1 \in (ty_2Elist_2Elist A_{\text{27a}}).(\forall V4l2 \in \\
 & (ty_2Elist_2Elist A_{\text{27a}}).(\forall V5l3 \in (ty_2Elist_2Elist A_{\text{27a}}). \\
 & (((ap (ap (c_2Elist_2EAPPEND A_{\text{27a}}) V4l2) V3l1) = (ap (ap (c_2Elist_2EAPPEND \\
 & A_{\text{27a}}) V5l3) V3l1)) \Leftrightarrow (V4l2 = V5l3))))))) \\
 & (43)
 \end{aligned}$$

Assume the following.

$(\forall V0n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge (\forall V2n \in ty_2Enum_2Enum. (\forall V3m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V3m) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2EiZ (ap (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge (\forall V4n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge (\forall V5n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge (\forall V6n \in ty_2Enum_2Enum. (\forall V7m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A V6n) V7m)))))) \wedge (\forall V8n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge (\forall V9n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge (\forall V10n \in ty_2Enum_2Enum. (\forall V11m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V10n) V11m)))))) \wedge (\forall V12n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V12n)))) = c_2Enum_2E0)) \wedge (\forall V13n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V13n)))) = c_2Enum_2E0)) \wedge (\forall V14n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge (\forall V15n \in ty_2Enum_2Enum. (\forall V16m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2EEEXP V15n) V16m)))))) \wedge (((ap c_2Enum_2ESUC c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge (\forall V17n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2ESUC V17n)))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V18n \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge (\forall V19n \in ty_2Enum_2Enum. (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge (\forall V20n \in ty_2Enum_2Enum. ((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge (\forall V21n \in ty_2Enum_2Enum. ((\forall V22m \in ty_2Enum_2Enum. (((ap c_2Earithmetic_2ENUMERAL V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge ((\forall V23n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V23n) c_2Enum_2E0)) \Leftrightarrow False))) \wedge (\forall V24n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V24n)))) \wedge (\forall V25n \in ty_2Enum_2Enum. (\forall V26m \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2ENUMERAL V25n) (ap c_2Earithmetic_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V27n)) \Leftrightarrow False))) \wedge (\forall V28n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V28n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3E c_2Earithmetic_2ENUMERAL V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V29n)) \Leftrightarrow True))) \wedge (\forall V30m \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V30m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3E c_2Earithmetic_2ENUMERAL V30m)))) \wedge ((\forall V31n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V31n)) \Leftrightarrow True))) \wedge (\forall V32n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V32n)) \Leftrightarrow True)))$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m)))))))))) \\
\end{aligned} \tag{46}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\\
& \forall V0l1 \in (ty_2Elist_2Elist A_27a). (\forall V1l2 \in (ty_2Elist_2Elist \\
& A_27a). (\forall V2f \in ((A_27b^{A_27a})^{ty_2Enum_2Enum}). ((ap (ap \\
& (c_2EindexedLists_2EMAPi A_27b A_27a) V2f) (ap (ap (c_2Elist_2EAPPEND \\
& A_27a) V0l1) V1l2)) = (ap (ap (c_2Elist_2EAPPEND A_27b) (ap (ap \\
& (c_2EindexedLists_2EMAPi A_27b A_27a) V2f) V0l1)) (ap (ap (c_2EindexedLists_2EMAPi \\
& A_27b A_27a) (ap (ap (c_2Ecombin_2Eo ty_2Enum_2Enum (A_27b^{A_27a}) \\
& ty_2Enum_2Enum) V2f) (ap c_2Earithmetic_2E_2B (ap (c_2Elist_2ELENGTH \\
& A_27a) V0l1)))) V1l2)))))) \\
\end{aligned}$$