

thm_2EindexedLists_2EfupdLast_EQ_NIL
(TMZjKNo-
qyh8DcsNwCpoVQwgbgELu1JXDQKP)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o$ ($p \ P \Rightarrow p \ Q$) of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap \ (ap \ (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c \in \text{Ebool} \cdot \text{E}_\lambda \cdot \text{E}_\lambda$ to be $\lambda A \cdot \text{E}_\lambda : \iota \cdot (\lambda V0P \in (2^{A \cdot \text{E}_\lambda}) \cdot (ap \ (ap \ (c \cdot \text{Emin} \cdot \text{E}_\lambda) \cdot \text{E}_\lambda) \cdot \text{E}_\lambda))$

Definition 5 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t\in 2.V0t))$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A0) \quad (1)$$

Let $c_2EindexedLists_2EupdLast : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2E\text{indexedLists_2E}f\text{updLast } A_27a \in (((ty_2Elist_2Elist } A_27a) \text{ } (ty_2Elist_2Elist } A_27a))^{(A_27a^{A_27a})}) \quad (2)$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p \ (ap \ P \ x))$ then $(the \ (\lambda x.x \in A \wedge p \ of \ type \ i \Rightarrow i))$

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A._27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A\ V0P)\ (c_2Ebool_2E_3F\ A\ V0P))\ V0P))$

Definition 9 We define $c_{\text{C}_2\text{Ebool_2E_5C_2F}}$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_{\text{C}_2\text{Ebool_2E_21}} 2))(\lambda V2t \in 2.(ap(c_{\text{C}_2\text{Ebool_2E_22}} 2))))$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (3)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (\text{ty_2Elist_2Elist } A_27a) \quad (4)$$

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(\text{ap } (\text{ap } c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E))$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2.(\text{False} \Rightarrow (p V0t))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge \text{True}) \Leftrightarrow (p V0t)) \wedge (((\text{False} \wedge (p V0t)) \Leftrightarrow \text{False}) \wedge (((p V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (8)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg \text{True}) \Leftrightarrow \text{False}) \wedge ((\neg \text{False}) \Leftrightarrow \text{True}))) \quad (9)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \text{True})) \quad (10)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow \text{True}) \Leftrightarrow (p V0t)) \wedge (((\text{False} \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(p V0t)))))) \quad (12)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0f \in (A_27a^{A_27a}).((\\
& \quad \text{ap } (\text{ap } (c_2EindexedLists_2EfupdLast } A_27a) V0f) (c_2Elist_2ENIL \\
& \quad A_27a)) = (c_2Elist_2ENIL A_27a))) \wedge ((\forall V1h \in A_27a.(\forall V2f \in \\
& \quad (A_27a^{A_27a}).((\text{ap } (\text{ap } (c_2EindexedLists_2EfupdLast } A_27a) V2f) \\
& \quad (\text{ap } (\text{ap } (c_2Elist_2ECONS } A_27a) V1h) (c_2Elist_2ENIL A_27a))) = \\
& \quad (\text{ap } (\text{ap } (c_2Elist_2ECONS } A_27a) (\text{ap } V2f V1h)) (c_2Elist_2ENIL A_27a)))) \wedge \\
& \quad (\forall V3v5 \in (ty_2Elist_2Elist } A_27a).(\forall V4v4 \in A_27a. \\
& \quad (\forall V5h \in A_27a.(\forall V6f \in (A_27a^{A_27a}).((\text{ap } (\text{ap } (c_2EindexedLists_2EfupdLast } \\
& \quad A_27a) V6f) (\text{ap } (\text{ap } (c_2Elist_2ECONS } A_27a) V5h) (\text{ap } (\text{ap } (c_2Elist_2ECONS } \\
& \quad A_27a) V4v4) V3v5))) = (\text{ap } (\text{ap } (c_2Elist_2ECONS } A_27a) V5h) (\text{ap } \\
& \quad (\text{ap } (c_2EindexedLists_2EfupdLast } A_27a) V6f) (\text{ap } (\text{ap } (c_2Elist_2ECONS } \\
& \quad A_27a) V4v4) V3v5)))))))))) \\
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist } \\
& \quad A_27a).((V0l = (c_2Elist_2ENIL A_27a)) \vee (\exists V1h \in A_27a.(\forall V2t \in \\
& \quad (ty_2Elist_2Elist } A_27a).((V0l = (\text{ap } (\text{ap } (c_2Elist_2ECONS } \\
& \quad A_27a) V1h) V2t)))))) \\
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0a1 \in (ty_2Elist_2Elist } \\
& \quad A_27a).(\forall V1a0 \in A_27a.(\neg((c_2Elist_2ENIL A_27a) = (\text{ap } \\
& \quad (\text{ap } (c_2Elist_2ECONS } A_27a) V1a0) V0a1)))))) \\
\end{aligned} \tag{15}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0f \in (A_27a^{A_27a}).(\forall V1x \in \\
& \quad (ty_2Elist_2Elist } A_27a).(((\text{ap } (\text{ap } (c_2EindexedLists_2EfupdLast } \\
& \quad A_27a) V0f) V1x) = (c_2Elist_2ENIL A_27a)) \Leftrightarrow (V1x = (c_2Elist_2ENIL \\
& \quad A_27a))) \wedge (((c_2Elist_2ENIL A_27a) = (\text{ap } (\text{ap } (c_2EindexedLists_2EfupdLast } \\
& \quad A_27a) V0f) V1x)) \Leftrightarrow (V1x = (c_2Elist_2ENIL A_27a)))))) \\
\end{aligned}$$