thm_2Einftree_2Einftree__nchotomy (TMSCxH5rCXSB4hmoRVxaqR8pqdN4LR4uWcz)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Definition 9 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let $c\_2Elist\_2ETL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETL\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (2)$$

Let $c\_2Elist\_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EHD\ A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (3)$$

Let $ty\_2Esum\_2Esum : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{4}$$

Let $ty\_2Einftree\_2Einftree : \iota\Rightarrow\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow\forall A2.nonempty\ A2\Rightarrow nonempty\ (ty\_2Einftree\_2Einftree\ A0\ A1\ A2) \tag{5}$$

Let $c\_2Einftree\_2Efrom\_\_inftree : \iota\Rightarrow\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27d.\ nonempty\ A\_27d\Rightarrow c\_2Einftree\_2Efrom\_\_inftree\ A\_27a\ A\_27b\ A\_27d \in (((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27d)})^{(ty\_2Einftree\_2Einftree\ A\_27a\ A\_27b\ A\_27d)}) \tag{6}$$

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Esum\_2EABS\_\_sum : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{7}$$

Definition 11 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Elist\_2ENIL : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{8}$$

Definition 12 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Einftree\_2Eto\_\_inftree : \iota\Rightarrow\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27d.\ nonempty\ A\_27d\Rightarrow c\_2Einftree\_2Eto\_\_inftree\ A\_27a\ A\_27b\ A\_27d \in ((ty\_2Einftree\_2Einftree\ A\_27a\ A\_27b\ A\_27d)^{((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27d)})}) \tag{9}$$

Definition 13 *We define* c_2Einftree_2EiNd *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0b \in A\_27b.\lambda V1f \in (($

Definition 14 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 15 *We define* c_2Einftree_2EiLf *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0a \in A\_27a.(ap\ (c\_2Ei$

Assume the following.

$$True \tag{10}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{17}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{18}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in A\_27a.(\exists V1x \in A\_27a.(V1x = V0a))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0a1 \in A\_27a.(\forall V1a2 \in A\_27a.(\forall V2b1 \in A\_27b.(\forall V3f1 \in ((ty\_2Einftree\_2Einftree\ A\_27a\ A\_27b\ A\_27c)^{A\_27c}).(\forall V4b2 \in A\_27b.(\forall V5f2 \in ((ty\_2Einftree\_2Einftree\ A\_27a\ A\_27b\ A\_27c)^{A\_27c}).((((ap\ (\mathsf{c\_2Einftree\_2EiLf}\ A\_27a\ A\_27b\ A\_27c)\ V0a1) = (ap\ (\mathsf{c\_2Einftree\_2EiLf}\ A\_27a\ A\_27b\ A\_27c)\ V1a2)) \Leftrightarrow (V0a1 = V1a2)) \wedge (((ap\ (ap\ (\mathsf{c\_2Einftree\_2EiNd}\ A\_27a\ A\_27b\ A\_27c)\ V2b1)\ V3f1) = (ap\ (ap\ (\mathsf{c\_2Einftree\_2EiNd}\ A\_27a\ A\_27b\ A\_27c)\ V4b2)\ V5f2)) \Leftrightarrow ((V2b1 = V4b2) \wedge (V3f1 = V5f2)))))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0a \in A\_27a.(\forall V1b \in A\_27b.(\forall V2f \in ((ty\_2Einftree\_2Einftree\ A\_27a\ A\_27b\ A\_27c)^{A\_27c}).(\neg((ap\ (\mathsf{c\_2Einftree\_2EiLf}\ A\_27a\ A\_27b\ A\_27c)\ V0a) = (ap\ (ap\ (\mathsf{c\_2Einftree\_2EiNd}\ A\_27a\ A\_27b\ A\_27c)\ V1b)\ V2f)))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0P \in (2^{(ty\_2Einftree\_2Einftree\ A\_27a\ A\_27b\ A\_27c)}).(((\forall V1a \in A\_27a.(p\ (ap\ V0P\ (ap\ (\mathsf{c\_2Einftree\_2EiLf}\ A\_27a\ A\_27b\ A\_27c)\ V1a)))) \wedge (\forall V2b \in A\_27b.(\forall V3f \in ((ty\_2Einftree\_2Einftree\ A\_27a\ A\_27b\ A\_27c)^{A\_27c}).((\forall V4d \in A\_27c.(p\ (ap\ V0P\ (ap\ V3f\ V4d)))) \Rightarrow (p\ (ap\ V0P\ (ap\ (ap\ (\mathsf{c\_2Einftree\_2EiNd}\ A\_27a\ A\_27b\ A\_27c)\ V2b)\ V3f))))))) \Rightarrow (\forall V5t \in (ty\_2Einftree\_2Einftree\ A\_27a\ A\_27b\ A\_27c).(p\ (ap\ V0P\ V5t))))) \tag{24}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0t \in (ty\_2Einftree\_2Einftree\ A\_27a\ A\_27b\ A\_27c).((\exists V1a \in A\_27a.(V0t = (ap\ (\mathsf{c\_2Einftree\_2EiLf}\ A\_27a\ A\_27b\ A\_27c)\ V1a))) \vee (\exists V2b \in A\_27b.(\exists V3d \in ((ty\_2Einftree\_2Einftree\ A\_27a\ A\_27b\ A\_27c)^{A\_27c}).(V0t = (ap\ (ap\ (\mathsf{c\_2Einftree\_2EiNd}\ A\_27a\ A\_27b\ A\_27c)\ V2b)\ V3d))))))$$