thm_2Einftree_2Erelrec__ind (TMNxpu-osu2sFBmQjXmPddyexyTZCdd2TfcQ)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2ETL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETL\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{2}$$

Let $c\_2Elist\_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EHD\ A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{3}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{4}$$

Let $ty\_2Einftree\_2Einftree : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow \forall A2.nonempty\ A2 \Rightarrow nonempty\ (ty\_2Einftree\_2Einftree\ A0\ A1\ A2) \tag{5}$$

Let $c\_2Einftree\_2Efrom\_\_inftree : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27d. nonempty\ A\_27d \Rightarrow c\_2Einftree\_2Efrom\_\_inftree\ A\_27a\ A\_27b\ A\_27d \in (((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27d)})^{(ty\_2Einftree\_2Einftree\ A\_27a\ A\_27b\ A\_27d)}) \quad (6)$$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \quad (7)$$

Definition 8 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS\_$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (8)$$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Einftree\_2Eto\_\_inftree : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27d. nonempty\ A\_27d \Rightarrow c\_2Einftree\_2Eto\_\_inftree\ A\_27a\ A\_27b\ A\_27d \in ((ty\_2Einftree\_2Einftree\ A\_27a\ A\_27b\ A\_27d)^{((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27d)})}) \quad (9)$$

Definition 11 *We define* c_2Einftree_2EiNd *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0b \in A\_27b.\lambda V1f \in (($

Definition 12 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 13 *We define* c_2Einftree_2EiLf *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0a \in A\_27a.(ap\ (c\_2Ei$

Definition 14 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 15 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 16 *We define* c_2Einftree_2Erelrec *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.(\lambda V0a0 \in (A$

Assume the following.

$$True \tag{10}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{12}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x)\Rightarrow(p\ V1y))\wedge((p\ V2z)\Rightarrow(p\ V3w)))\Rightarrow(((p\ V0x)\wedge(p\ V2z))\Rightarrow((p\ V1y)\wedge(p\ V3w)))))))) \tag{13}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x)\Rightarrow(p\ V1y))\wedge((p\ V2z)\Rightarrow(p\ V3w)))\Rightarrow(((p\ V0x)\vee(p\ V2z))\Rightarrow((p\ V1y)\vee(p\ V3w)))))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x))\Rightarrow(p\ (ap\ V1Q\ V2x))))\Rightarrow((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x)))\Rightarrow(\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x))\Rightarrow(p\ (ap\ V1Q\ V2x))))\Rightarrow((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x)))\Rightarrow(\exists V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{16}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.$$
$$nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0relrec\_27 \in$$
$$((((2^{A\_27b})^{(ty\_2Einftree\_2Einftree\ A\_27a\ A\_27c\ A\_27d)})^{((A\_27b^{(A\_27b^{A\_27d})})^{A\_27c})})^{(A\_27b^{A\_27a})}).$$
$$(((\forall V1lf \in (A\_27b^{A\_27a}).(\forall V2nd \in ((A\_27b^{(A\_27b^{A\_27d})})^{A\_27c}).$$
$$(\forall V3a \in A\_27a.(p\ (ap\ (ap\ (ap\ (ap\ V0relrec\_27\ V1lf)\ V2nd)\ (ap$$
$$(\mathsf{c\_2Einftree\_2EiLf}\ A\_27a\ A\_27c\ A\_27d)\ V3a))\ (ap\ V1lf\ V3a))))))\wedge$$
$$(\forall V4lf \in (A\_27b^{A\_27a}).(\forall V5nd \in ((A\_27b^{(A\_27b^{A\_27d})})^{A\_27c}).$$
$$(\forall V6b \in A\_27c.(\forall V7df \in ((ty\_2Einftree\_2Einftree$$
$$A\_27a\ A\_27c\ A\_27d)^{A\_27d}).(\forall V8g \in (A\_27b^{A\_27d}).((\forall V9d \in$$
$$A\_27d.(p\ (ap\ (ap\ (ap\ (ap\ V0relrec\_27\ V4lf)\ V5nd)\ (ap\ V7df\ V9d))\ (ap$$
$$V8g\ V9d)))) \Rightarrow (p\ (ap\ (ap\ (ap\ (ap\ V0relrec\_27\ V4lf)\ V5nd)\ (ap\ (ap\ (\mathsf{c\_2Einftree\_2EiNd}$$
$$A\_27a\ A\_27c\ A\_27d)\ V6b)\ V7df))\ (ap\ (ap\ V5nd\ V6b)\ V8g)))))))))) \Rightarrow ($$
$$\forall V10a0 \in (A\_27b^{A\_27a}).(\forall V11a1 \in ((A\_27b^{(A\_27b^{A\_27d})})^{A\_27c}).$$
$$(\forall V12a2 \in (ty\_2Einftree\_2Einftree\ A\_27a\ A\_27c\ A\_27d).($$
$$\forall V13a3 \in A\_27b.((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Einftree\_2Erelrec}$$
$$A\_27a\ A\_27b\ A\_27c\ A\_27d)\ V10a0)\ V11a1)\ V12a2)\ V13a3)) \Rightarrow (p\ (ap\ (ap$$
$$(ap\ (ap\ V0relrec\_27\ V10a0)\ V11a1)\ V12a2)\ V13a3)))))))))$$