

thm_2Einfree_2Erelrec_rules

(TMMHQbhgwGiqvdP3wmxfJz3SNz3oUjv86Ux)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2EF$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2E_2ETL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2E_2ETL A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)}) \quad (2)$$

Let $c_2Elist_2E_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2E_2EHD A_27a \in (A_27a^{(ty_2Elist_2Elist A_27a)}) \quad (3)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (4)$$

Let $ty_2Einfree_2Einfree : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow \forall A2.nonempty A2 \Rightarrow nonempty (ty_2Einfree_2Einfree A0 A1 A2) \quad (5)$$

Let $c_2Einf_tree_2Efrom_inf_tree : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27d. \\ & nonempty\ A_27d \Rightarrow c_2Einf_tree_2Efrom_inf_tree\ A_27a\ A_27b\ A_27d \in \\ & ((ty_2Esum_2Esum\ A_27a\ A_27b)^{(ty_2Elist_2Elist\ A_27d)}(ty_2Einf_tree_2Einf_tree\ A_27a\ A_27b\ A_27d)) \end{aligned} \quad (6)$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ & A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (7)$$

Definition 8 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist \\ & A_27a) \end{aligned} \quad (8)$$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\mathbf{if}$

Let $c_2Einf_tree_2Eto_inf_tree : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27d. \\ & nonempty\ A_27d \Rightarrow c_2Einf_tree_2Eto_inf_tree\ A_27a\ A_27b\ A_27d \in \\ & ((ty_2Einf_tree_2Einf_tree\ A_27a\ A_27b\ A_27d)^{(ty_2Esum_2Esum\ A_27a\ A_27b)^{(ty_2Elist_2Elist\ A_27d)}}) \end{aligned} \quad (9)$$

Definition 11 We define $c_2Einf_tree_2EiNd$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0b \in A_27b.\lambda V1f \in (($

Definition 12 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum$

Definition 13 We define $c_2Einf_tree_2EiLf$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0a \in A_27a.(ap\ (c_2Einf_tree$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 15 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Definition 16 We define $c_2Einf_tree_2Erelrec$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.(\lambda V0a0 \in (A$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w))) \Rightarrow (((p V0x) \wedge (p V2z)) \Rightarrow ((p V1y) \wedge (p V3w)))))) \quad (13)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w))) \Rightarrow (((p V0x) \vee (p V2z)) \Rightarrow ((p V1y) \vee (p V3w)))))) \quad (14)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \Rightarrow (p (ap V1Q V2x)))) \Rightarrow ((\forall V3x \in A.27a.(p (ap V0P V3x))) \Rightarrow (\forall V4x \in A.27a.(p (ap V1Q V4x)))))) \quad (15)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \Rightarrow (p (ap V1Q V2x)))) \Rightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \Rightarrow (\exists V4x \in A.27a.(p (ap V1Q V4x)))))) \quad (16)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\ & \text{nonempty } A_27c \Rightarrow \forall A_27d.\text{nonempty } A_27d \Rightarrow ((\forall V0lf \in \\ & (A_27b^{A_27a}).(\forall V1nd \in ((A_27b^{(A_27b^{A_27d})})^{A_27c}).(\\ & \forall V2a \in A_27a.(p (ap (ap (ap (ap (c_2Einf tree_2Erelrec } A_27a \\ & A_27b } A_27c } A_27d) } V0lf) } V1nd) (ap (c_2Einf tree_2EiLf } A_27a } A_27c \\ & A_27d) } V2a)) (ap } V0lf } V2a)))))) \wedge (\forall V3lf \in (A_27b^{A_27a}).(\\ & \forall V4nd \in ((A_27b^{(A_27b^{A_27d})})^{A_27c}).(\forall V5b \in A_27c. \\ & (\forall V6df \in ((ty_2Einf tree_2Einf tree } A_27a } A_27c } A_27d)^{A_27d}). \\ & (\forall V7g \in (A_27b^{A_27d}).((\forall V8d \in A_27d.(p (ap (ap (ap \\ & (ap (c_2Einf tree_2Erelrec } A_27a } A_27b } A_27c } A_27d) } V3lf) } V4nd) \\ & (ap } V6df } V8d)) (ap } V7g } V8d)))))) \Rightarrow (p (ap (ap (ap (ap (c_2Einf tree_2Erelrec \\ & A_27a } A_27b } A_27c } A_27d) } V3lf) } V4nd) (ap (ap (c_2Einf tree_2EiNd \\ & A_27a } A_27c } A_27d) } V5b) } V6df)) (ap (ap } V4nd } V5b) } V7g))))))))) \end{aligned}$$