thm_2Einftree_2Erelrec__strongind (TMcXhE7XMas39BYj9E5gbPc1j9gKNB9y5EP)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj__o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^{A_27a}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj__o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c_2Emin_2E_3D_3D_3E}\ V0t)\ \mathsf{c_2Ebool_2EF}$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{1}$$

Let $c_2Elist_2ETL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ETL\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \tag{2}$$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EHD\ A_27a \in (A_27a^{(ty_2Elist_2Elist\ A_27a)}) \tag{3}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{4}$$

Let $ty_2Einftree_2Einftree : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow \forall A2.nonempty\ A2 \Rightarrow nonempty\ (ty_2Einftree_2Einftree\ A0\ A1\ A2) \tag{5}$$

Let $c_2Einftree_2Efrom__inftree : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27d.\ nonempty\ A_27d \Rightarrow c_2Einftree_2Efrom__inftree\ A_27a\ A_27b\ A_27d \in (((ty_2Esum_2Esum\ A_27a\ A_27b)^{(ty_2Elist_2Elist\ A_27d)})^{(ty_2Einftree_2Einftree\ A_27a\ A_27b\ A_27d)}) \tag{6}$$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in 2$

Let $c_2Esum_2EABS__sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS__sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{(((2^{A_27b})^{A_27a})^2)}) \tag{7}$$

Definition 8 *We define* c_2Esum_2EINR *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \tag{8}$$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2ECOND *to be* $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Let $c_2Einftree_2Eto__inftree : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27d.\ nonempty\ A_27d \Rightarrow c_2Einftree_2Eto__inftree\ A_27a\ A_27b\ A_27d \in ((ty_2Einftree_2Einftree\ A_27a\ A_27b\ A_27d)^{((ty_2Esum_2Esum\ A_27a\ A_27b)^{(ty_2Elist_2Elist\ A_27d)})}) \tag{9}$$

Definition 11 *We define* c_2Einftree_2EiNd *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0b \in A_27b.\lambda V1f \in ((($

Definition 12 *We define* c_2Esum_2EINL *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Definition 13 *We define* c_2Einftree_2EiLf *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0a \in A_27a.(ap\ (c_2Ei$

Definition 14 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (\mathsf{c_2Emin_2E_40}$

Definition 15 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in$

Definition 16 *We define* c_2Einftree_2Erelrec *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.(\lambda V0a0 \in (A$

Assume the following.

$$True \tag{10}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{12}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w)))))))) \tag{13}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow ((p\ V1y) \vee (p\ V3w)))))))) \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\forall V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\forall V4x \in A_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\exists V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{16}$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.$$
$$nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0relrec_27 \in$$
$$((((2^{A_27b})^{(ty_2Einftree_2Einftree\ A_27a\ A_27c\ A_27d)})^{((A_27b^{(A_27b^{A_27d})})^{A_27c})})^{(A_27b^{A_27a})}).$$
$$(((\forall V1lf \in (A_27b^{A_27a}).(\forall V2nd \in ((A_27b^{(A_27b^{A_27d})})^{A_27c}).$$
$$(\forall V3a \in A_27a.(p\ (ap\ (ap\ (ap\ (ap\ V0relrec_27\ V1lf)\ V2nd)\ (ap$$
$$(\mathsf{c_2Einftree_2EiLf}\ A_27a\ A_27c\ A_27d)\ V3a))\ (ap\ V1lf\ V3a)))))) \wedge$$
$$(\forall V4lf \in (A_27b^{A_27a}).(\forall V5nd \in ((A_27b^{(A_27b^{A_27d})})^{A_27c}).$$
$$(\forall V6b \in A_27c.(\forall V7df \in ((ty_2Einftree_2Einftree$$
$$A_27a\ A_27c\ A_27d)^{A_27d}).(\forall V8g \in (A_27b^{A_27d}).((\forall V9d \in$$
$$A_27d.((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c_2Einftree_2Erelrec}\ A_27a\ A_27b\ A_27c$$
$$A_27d)\ V4lf)\ V5nd)\ (ap\ V7df\ V9d))\ (ap\ V8g\ V9d))) \wedge (p\ (ap\ (ap\ (ap\ (ap$$
$$V0relrec_27\ V4lf)\ V5nd)\ (ap\ V7df\ V9d))\ (ap\ V8g\ V9d))))) \Rightarrow (p\ (ap\ (ap$$
$$(ap\ (ap\ V0relrec_27\ V4lf)\ V5nd)\ (ap\ (ap\ (\mathsf{c_2Einftree_2EiNd}\ A_27a$$
$$A_27c\ A_27d)\ V6b)\ V7df))\ (ap\ (ap\ V5nd\ V6b)\ V8g))))))))) \Rightarrow (\forall V10a0 \in$$
$$(A_27b^{A_27a}).(\forall V11a1 \in ((A_27b^{(A_27b^{A_27d})})^{A_27c}).$$
$$(\forall V12a2 \in (ty_2Einftree_2Einftree\ A_27a\ A_27c\ A_27d).($$
$$\forall V13a3 \in A_27b.((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c_2Einftree_2Erelrec}$$
$$A_27a\ A_27b\ A_27c\ A_27d)\ V10a0)\ V11a1)\ V12a2)\ V13a3)) \Rightarrow (p\ (ap\ (ap$$
$$(ap\ (ap\ V0relrec_27\ V10a0)\ V11a1)\ V12a2)\ V13a3)))))))))$$