

thm_2Eint_arith_2EINT_NUM_DIVIDES
 (TMEttR7LpdZQnc5G1YXjBsRbBEF5noXFppf)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (3)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (4)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (5)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num (m))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (6)$$

Definition 5 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP).$

Definition 6 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1 n) 0)$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p x) \text{ of type } \iota \Rightarrow \iota)$

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a) P)))$

Definition 11 We define $c_2Edivides_2Edivides$ to be $\lambda V0a \in ty_2Enum_2Enum.\lambda V1b \in ty_2Enum_2Enum.(c_2Edivides (ap (c_2Emin_2E_40 (ty_2Enum_2Enum a))) b))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty } (ty_2Epair_2Eprod \\ A0 A1) \end{aligned} \quad (8)$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Einteger_2Eint \quad (9)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Einteger_2Eint_REP_CLASS}) \quad (10)$$

Definition 12 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum a)))$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (11)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (12)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}} \quad (13)$$

Definition 13 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum).(\lambda V1s \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum r))) s))$

Definition 14 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum T1))) T2))$

Definition 15 We define $c_2Einteger_2Eint_divides$ to be $\lambda V0p \in ty_2Einteger_2Eint.\lambda V1q \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum p))) q))$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2 \in \text{integer} \cdot \text{tint_neg} \in ((\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum} \\ \text{ty_2Enum_2Enum})^{(\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum} \text{ ty_2Enum_2Enum})}) \quad (14)$$

Definition 16 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint_neg\ V0)$

Let $c.2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (15)$$

Definition 17 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t\ t \in 2.V0t))$.

Definition 18 We define $c_2 \in \text{Emin_3D_3E}$ to be $\lambda P \in 2.\lambda Q \in 2.\text{inj_o} (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 19 We define $c_{\text{Ebool_2E_5C_2F}}$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_{\text{Ebool_2E_21}} 2) (\lambda V2t \in$

Definition 20 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21 2))(\lambda V2t \in$

Definition 21 We define $c_2.Ebool_E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2.Emin_2E_3D_3D_3E\ V0t)\ c_2.Ebool_2E)$

Assume the following.

($\forall V \exists m \in t$)

$((ap (ap c_2Earithmetic_2E_2A c_2Enum_2EU) V0m) = c_2Enum_2EU) \wedge$
 $((((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge$
 $((((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL$
 $(ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge$
 $((((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL$
 $(ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge ($
 $((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap$
 $(ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n))$
 $V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) =$
 $(ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A$
 $V0m) V1n)))))))))))$

Assume the following.

True (17)

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & \quad ((p V0t) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & \quad (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & \quad (p V0t)) \Leftrightarrow (p V0t))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & \quad True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & \quad (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \end{aligned} \quad (25)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))) \quad (26)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ True)) \quad (27)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & \quad ((p V0t) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & \quad (p V0t))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\forall A_{\leq 27a}. \text{nonempty } A_{\leq 27a} \Rightarrow (\forall V0P \in (2^{A_{\leq 27a}}). ((\neg(\exists V1x \in A_{\leq 27a}. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_{\leq 27a}. (\neg(p (ap V0P V2x))))) \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A_{\leq 27a}. \text{nonempty } A_{\leq 27a} \Rightarrow & (\forall V0P \in (2^{A_{\leq 27a}}). (\forall V1Q \in (2^{A_{\leq 27a}}). ((\forall V2x \in A_{\leq 27a}. ((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A_{\leq 27a}. (p (ap V0P V3x))) \wedge (\forall V4x \in A_{\leq 27a}. (p (ap V1Q V4x))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_{\leq 27a}. \text{nonempty } A_{\leq 27a} \Rightarrow & (\forall V0P \in (2^{A_{\leq 27a}}). (\forall V1Q \in (2^{A_{\leq 27a}}). \\ 2. & ((\forall V2x \in A_{\leq 27a}. ((p (ap V0P V2x)) \Rightarrow (p V1Q))) \Leftrightarrow ((\exists V3x \in A_{\leq 27a}. (p (ap V0P V3x)) \Rightarrow (p V1Q)))))) \end{aligned} \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (37)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1))))) \quad (38)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x_{\leq 27} \in 2. (\forall V2y \in 2. (\forall V3y_{\leq 27} \in 2. \\ 2. & (((p V0x) \Leftrightarrow (p V1x_{\leq 27})) \wedge ((p V1x_{\leq 27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{\leq 27})))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{\leq 27}) \Rightarrow (p V3y_{\leq 27}))))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0f \in (2^{A_27a}).(\forall V1v \in \\ A_27a.((\forall V2x \in A_27a.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (\\ ap V0f V1v)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ ((ap c_2Einteger_2Eint_of_num V0m) = (ap c_2Einteger_2Eint_of_num \\ V1n)) \Leftrightarrow (V0m = V1n)))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ (ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num \\ V0m)) (ap c_2Einteger_2Eint_of_num V1n)) = (ap c_2Einteger_2Eint_of_num \\ (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in ty_2Einteger_2Eint.((\exists V1n \in ty_2Enum_2Enum. \\ ((V0p = (ap c_2Einteger_2Eint_of_num V1n)) \wedge (\neg(V1n = c_2Enum_2E0)))) \vee \\ ((\exists V2n \in ty_2Enum_2Enum.((V0p = (ap c_2Einteger_2Eint_neg \\ (ap c_2Einteger_2Eint_of_num V2n))) \wedge (\neg(V2n = c_2Enum_2E0)))) \vee \\ (V0p = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} ((\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\ ((ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num \\ V0m)) (ap c_2Einteger_2Eint_of_num V1n)) = (ap c_2Einteger_2Eint_of_num \\ (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))) \wedge ((\forall V2x \in ty_2Einteger_2Eint. \\ (\forall V3y \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_mul \\ (ap c_2Einteger_2Eint_neg V2x)) V3y) = (ap c_2Einteger_2Eint_neg \\ (ap (ap c_2Einteger_2Eint_mul V2x) V3y)))))) \wedge ((\forall V4x \in ty_2Einteger_2Eint. \\ (\forall V5y \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_mul \\ (ap c_2Einteger_2Eint_neg V5y)) = (ap c_2Einteger_2Eint_neg \\ (ap (ap c_2Einteger_2Eint_mul V4x) V5y)))))) \wedge ((\forall V6x \in ty_2Einteger_2Eint. \\ ((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V6x)) = \\ V6x))))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& (((ap c_2Einteger_2Eint_of_num V0m) = (ap c_2Einteger_2Eint_of_num \\
& V1n)) \Leftrightarrow (V0m = V1n))) \wedge (\forall V2x \in ty_2Einteger_2Eint. (\forall V3y \in \\
& ty_2Einteger_2Eint. (((ap c_2Einteger_2Eint_neg V2x) = (ap c_2Einteger_2Eint_neg \\
& V3y)) \Leftrightarrow (V2x = V3y)))) \wedge (\forall V4n \in ty_2Enum_2Enum. (\forall V5m \in \\
& ty_2Enum_2Enum. (((ap c_2Einteger_2Eint_of_num V4n) = (ap \\
& c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V5m))) \Leftrightarrow \\
& ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))) \wedge (((ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V4n)) = (ap c_2Einteger_2Eint_of_num \\
& V5m)) \Leftrightarrow ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))))))) \\
& \tag{45}
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{46}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
& \tag{48}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
& \tag{49}
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p \\
& V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \\
& \tag{51}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
& \tag{52}
\end{aligned}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (54)$$

Theorem 1

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (p (ap (ap c_2Einteger_2Eint_divides (ap c_2Einteger_2Eint_of_num V0n) (ap c_2Einteger_2Eint_of_num V1m))) \Leftrightarrow (p (ap (ap c_2Edivides_2Edivides V0n) V1m)))))$$